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WHEN THE PROPERTY OF HAVING A π-TREE IS PRESERVED BY PRODUCTS

MIKHAIL PATRAKEEV

Abstract. We find sufficient conditions under which the product of spaces that have a π-tree also has a π-tree. These conditions give new examples of spaces with a π-tree: every at most countable power of the Sorgenfrey line and every at most countable power of the irrational Sorgenfrey line has a π-tree. Also we show that if a space has a π-tree, then its product with the Baire space, with the Sorgenfrey line, and with the countable power of the Sorgenfrey line also has a π-tree.

1. Introduction

We study topological spaces that have a π-tree, see Terminology 2.5 in §2. The notion of a π-tree was introduced in [10] and is equivalent [10, Remark 11] to the notion of a Lusin π-base, which was introduced in [8]. The Sorgenfrey line $\mathcal{R}_S$ and the Baire space $\mathcal{N}$ (that is, ${}^{\omega}\omega$ with the product topology) are examples of spaces with a π-tree [8]. Every space that has a π-tree shares many good properties with the Baire space. One reason for this is expressed in Lemma 2.6 and Lemma 3.2; another two are the following: If a space X has a π-tree, then X can be mapped onto $\mathcal{N}$ by a continuous one-to-one map [8] and also X can be mapped onto $\mathcal{N}$ by a continuous open map [8] (hence, X can be mapped by a continuous open map onto an arbitrary Polish space, see [1] or [6, Exercise 7.14]). Every space that has a π-tree also has a countable π-base, see Lemma 2.7.

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