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ON BOREL SEMIFILTERS

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Abstract. Building on work of van Engelen and van Mill, we show that a zero-dimensional Borel space is homeomorphic to a semifilter if and only if it is homogeneous and not locally compact. Under $\mathbf{\Sigma}^1_1$-Determinacy, this result extends to all analytic and coanalytic spaces.

1. Introduction

Throughout this paper, Ω will denote a countably infinite set. We will denote by $\mathcal{P}(\Omega)$ the collection of all subsets of Ω. Define $\mathsf{Fin}(\Omega) = \{x \subseteq \Omega : x \text{ is finite}\}$ and $\mathsf{Cof}(\Omega) = \{x \subseteq \Omega : \Omega \setminus x \text{ is finite}\}$. Also define $\mathsf{Fin} = \mathsf{Fin}(\omega)$ and $\mathsf{Cof} = \mathsf{Cof}(\omega)$.

A collection $\mathcal{X} \subseteq \mathcal{P}(\Omega)$ is *upward-closed* if and only if $y \supseteq x \in \mathcal{X}$ implies $y \in \mathcal{X}$ for all $x, y \in \mathcal{P}(\Omega)$. We will write $x \subseteq^* y$ to mean that $x \setminus y$ is finite, and we will write $x =^* y$ to mean that $x \subseteq^* y$ and $y \subseteq^* x$. A collection $\mathcal{X} \subseteq \mathcal{P}(\Omega)$ is *closed under finite modifications* if and only if $y =^* x \in \mathcal{X}$ implies $y \in \mathcal{X}$ for all $x, y \in \mathcal{P}(\Omega)$.

A *semifilter on* Ω is a collection $\mathcal{S} \subseteq \mathcal{P}(\Omega)$ that satisfies the following conditions.

- $\varnothing \notin \mathcal{S}$ and $\Omega \in \mathcal{S}$.
- $\mathcal{S}$ is closed under finite modifications.
- $\mathcal{S}$ is upward-closed.

All semifilters are assumed to be on ω unless we explicitly say otherwise.

The notion of semifilter is a natural weakening of the notion of filter, and it has found applications in several areas of mathematics (see [1]).

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