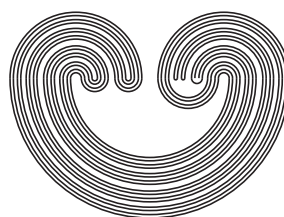


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## WEAK SELECTIONS AND COUNTABLE COMPACTNESS

by

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## WEAK SELECTIONS AND COUNTABLE COMPACTNESS

KOICHI MOTOOKA

**ABSTRACT.** We prove that every countably compact Hausdorff space with a continuous weak selection is weakly orderable, which answers a question of Buhagiar and Gutev affirmatively. We also prove that every feebly compact regular space with a continuous weak selection is suborderable.

### 1. INTRODUCTION

All spaces in this paper are assumed to be Hausdorff topological spaces. For a space  $X$ , let  $\mathcal{F}_2(X) = \{F \subset X : 1 \leq |F| \leq 2\}$ , where  $|F|$  is the cardinality of  $F$ . The set  $\mathcal{F}_2(X)$  is assumed to have the *Vietoris topology*  $\tau_{\mathcal{V}}$  which has a base consisting of all sets of the form

$$\langle \mathcal{V} \rangle = \{S \in \mathcal{F}_2(X) : S \subset \bigcup \mathcal{V} \text{ and } S \cap V \neq \emptyset \text{ for each } V \in \mathcal{V}\},$$

where  $\mathcal{V}$  runs over all finite families of open subsets of  $X$ . (It suffices to take only  $|\mathcal{V}| \leq 2$  here.) We say that a function  $\sigma : \mathcal{F}_2(X) \rightarrow X$  is a *weak selection* on the space  $X$  if  $\sigma(F) \in F$  for every  $F \in \mathcal{F}_2(X)$ . A weak selection on the space  $X$  is said to be *continuous* if it is continuous with respect to the Vietoris topology on  $\mathcal{F}_2(X)$  and the topology of  $X$ .

For a linear order  $\preceq$  on a set  $X$ , let  $\tau_{\preceq}$  be the order topology generated by  $\preceq$ . A space  $(X, \tau)$  is *orderable* (respectively, *weakly orderable*) if  $\tau_{\preceq} = \tau$  (respectively,  $\tau_{\preceq} \subset \tau$ ) for some linear ordering  $\preceq$  on the set  $X$ .

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*Key words and phrases.* Vietoris topology, continuous weak selection, locally uniform weak selection, countably compact, sequentially compact, weakly orderable, suborderable.

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