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by

Koichi Motooka

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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WEAK SELECTIONS AND COUNTABLE COMPACTNESS

KOICHI MOTOOKA

Abstract. We prove that every countably compact Hausdorff space with a continuous weak selection is weakly orderable, which answers a question of Buhagiar and Gutev affirmatively. We also prove that every feebly compact regular space with a continuous weak selection is suborderable.

1. Introduction

All spaces in this paper are assumed to be Hausdorff topological spaces. For a space X, let $\mathcal{F}_2(X) = \{F \subset X : 1 \leq |F| \leq 2\}$, where $|F|$ is the cardinality of F. The set $\mathcal{F}_2(X)$ is assumed to have the *Vietoris topology* τ_V which has a base consisting of all sets of the form

$$\langle \mathcal{V} \rangle = \{S \in \mathcal{F}_2(X) : S \subset \bigcup \mathcal{V} \text{ and } S \cap V \neq \emptyset \text{ for each } V \in \mathcal{V}\},$$

where $\mathcal{V}$ runs over all finite families of open subsets of X. (It suffices to take only $|\mathcal{V}| \leq 2$ here.) We say that a function $\sigma : \mathcal{F}_2(X) \to X$ is a *weak selection* on the space X if $\sigma(F) \in F$ for every $F \in \mathcal{F}_2(X)$. A weak selection on the space X is said to be *continuous* if it is continuous with respect to the Vietoris topology on $\mathcal{F}_2(X)$ and the topology of X.

For a linear order $\preceq$ on a set X, let $\tau_{\preceq}$ be the order topology generated by $\preceq$. A space (X, τ) is *orderable* (respectively, *weakly orderable*) if $\tau_{\preceq} = \tau$ (respectively, $\tau_{\preceq} \subset \tau$) for some linear ordering $\preceq$ on the set X.

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