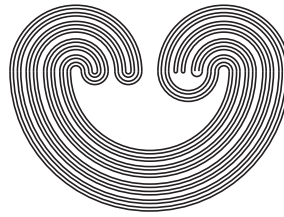


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by

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## SOME TOPOLOGICAL GAMES, $D$ -SPACES AND COVERING PROPERTIES OF HYPERSPACES

LIANG-XUE PENG\*, YUAN SUN, AND SHANG-ZHI WANG

**ABSTRACT.** We study topological games,  $D$ -spaces and covering properties of hyperspaces with the upper (lower) Vietoris topology  $V^+(V^-)$ . Let  $\mathbf{C}$  be the class of all compact spaces. Let  $\mathbf{W}$  be the class of all countable spaces. Let  $\mathbf{1}$  denote the class of all one point spaces and empty set. We get the following conclusions:

If  $X$  is a  $\mathbf{1}$ -like  $T_1$ -space, then the hyperspace  $(2^X, V^-)$   $((\mathcal{C}(X), V^-))$  is a  $\mathbf{1}$ -like space. If  $X$  is a  $nc\text{-}\mathbf{W}$ -like  $T_1$ -space, then  $(2^X, V^-)$   $((\mathcal{C}(X), V^-))$  is a  $nc\text{-}\mathbf{W}$ -like space. If  $X$  is a  $\mathbf{D1}$ -like  $T_1$ -space, then  $(2^X, V^-)$   $((\mathcal{C}(X), V^-))$  is a  $\mathbf{D1}$ -like space. If  $X$  is a  $T_1$ -space and  $(\mathcal{C}(X), V^+)$  is  $nc\text{-}\mathbf{1}$ -like, then  $X$  is  $\mathbf{C}$ -like. If  $X$  is a  $T_1$ -space and  $(\mathcal{C}(X), V^-)$  is  $nc\text{-}\mathbf{1}$ -like, then  $X$  is  $\mathbf{C}$ -like. If  $X$  is a hemicompact Hausdorff space, then  $(\mathcal{C}(X), V^+)$  is a  $nc\text{-}\mathbf{1}$ -like space. We finally show that if  $X$  is a Hausdorff topological space such that every closed compact subset of  $X$  is a  $G_\delta$ -set of  $X$ , then  $(\mathcal{C}(X), V^+)$  is a  $nc\text{-}\mathbf{1}$ -like space if and only if  $X$  is a hemicompact space. We point out that there exists a  $\sigma$ -compact ( $\mathbf{1}$ -like)  $T_2$ -space  $X$  such that the hyperspace  $(\mathcal{C}(X), V^+)$  is not a  $nc\text{-}\mathbf{1}$ -like space.

If  $X$  is a  $T_1$   $D$ -space, then  $(2^X, V^-)$  is a  $D$ -space. If  $X$  is a  $T_1$ -space, then  $X$  is a  $D$ -space if and only if  $(\mathcal{C}(X), V^-)$  is a  $D$ -space. If  $X$  is a  $T_1$ -space and  $(2^X, V^-)$  is a  $bD$ -space, then  $X$  is a  $bD$ -space. If  $X$  is a paracompact space, then  $(2^X, V^-)$  is metacompact. If  $X_n$  is a  $T_1$ -space for each  $n \in \mathbb{N}$  such that  $\prod_{n \in \mathbb{N}} (\mathcal{C}(X_n), V_n^+)$  is Lindelöf, then  $\prod_{n \in \mathbb{N}} X_n$  is Lindelöf.

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*Key words and phrases.* Topological games,  $\mathbf{C}$ -like,  $\mathbf{1}$ -like,  $\mathbf{W}$ -like,  $D$ -space, metacompact, hemicompact, hyperspace.

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