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SOME TOPOLOGICAL GAMES, D-SPACES AND COVERING PROPERTIES OF HYPERSPACES

LIANG-XUE PENG*, YUAN SUN, AND SHANG-ZHI WANG

Abstract. We study topological games, D-spaces and covering properties of hyperspaces with the upper (lower) Vietoris topology $V^+(V^-)$. Let $\mathbf{C}$ be the class of all compact spaces. Let $\mathbf{W}$ be the class of all countable spaces. Let $\mathbf{1}$ denote the class of all one point spaces and empty set. We get the following conclusions:

If X is a $\mathbf{1}$-like T_1-space, then the hyperspace $(2^X, V^-)$ $((\mathcal{C}(X), V^-))$ is a $\mathbf{1}$-like space. If X is a nc-$\mathbf{W}$-like T_1-space, then $(2^X, V^-)$ $((\mathcal{C}(X), V^-))$ is a nc-$\mathbf{W}$-like space. If X is a $\mathbf{D1}$-like T_1-space, then $(2^X, V^-)$ $((\mathcal{C}(X), V^-))$ is a $\mathbf{D1}$-like space. If X is a T_1-space and $(\mathcal{C}(X), V^+)$ is nc-$\mathbf{1}$-like, then X is $\mathbf{C}$-like. If X is a T_1-space and $(\mathcal{C}(X), V^-)$ is nc-$\mathbf{1}$-like, then X is $\mathbf{C}$-like. If X is a hemicompact Hausdorff space, then $(\mathcal{C}(X), V^+)$ is a nc-$\mathbf{1}$-like space. We finally show that if X is a Hausdorff topological space such that every closed compact subset of X is a G_δ-set of X, then $(\mathcal{C}(X), V^+)$ is a nc-$\mathbf{1}$-like space if and only if X is a hemicompact space. We point out that there exists a σ-compact ($\mathbf{1}$-like) T_2-space X such that the hyperspace $(\mathcal{C}(X), V^+)$ is not a nc-$\mathbf{1}$-like space.

If X is a T_1 D-space, then $(2^X, V^-)$ is a D-space. If X is a T_1-space, then X is a D-space if and only if $(\mathcal{C}(X), V^-)$ is a D-space. If X is a T_1-space and $(2^X, V^-)$ is a bD-space, then X is a bD-space. If X is a paracompact space, then $(2^X, V^-)$ is metacompact. If X_n is a T_1-space for each $n \in \mathbb{N}$ such that $\prod_{n\in\mathbb{N}} (\mathcal{C}(X_n), V_n^+)$ is Lindelöf, then $\prod_{n\in\mathbb{N}} X_n$ is Lindelöf.

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Key words and phrases. Topological games, $\mathbf{C}$-like, $\mathbf{1}$-like, $\mathbf{W}$-like, D-space, metacompact, hemicompact, hyperspace.

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