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TOPOLOGY PROCEEDINGS

Volume 53, 2019

Pages 301-317

http://topology.nipissingu.ca/tp/

Hereditary paracompactness of lexicographic products

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Electronically published on January 31, 2019

Topology Proceedings

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ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 53 (2019)
Pages 301-317

E-Published on January 31, 2019

HEREDITARY PARACOMPACTNESS OF LEXICOGRAPHIC PRODUCTS

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Abstract. Paracompactness and hereditary paracompactness of lexicographic products of LOTS's are discussed in [2]. For instance, it is known in [2]:

- a lexicographic product $X = \prod_{\alpha<\gamma} X_\alpha$ of LOTS's is paracompact whenever all X_α's are paracompact [2, Theorem 4.2.2],
- a lexicographic product $X = \prod_{\alpha<\gamma} X_\alpha$ of LOTS's is hereditarily paracompact whenever $\gamma < \omega_1$ and all X_α's are hereditarily paracompact [2, Theorem 4.2.3],
- the lexicographic product $[0,1]_{\mathbb{R}}^{\omega_1}$ is not hereditarily paracompact, where $[0,1]_{\mathbb{R}}$ denotes the unit interval in the real line $\mathbb{R}$ [2, page 73].

Recently the author defined the notion of lexicographic products of GO-spaces and extended the first result above in [2] for lexicographic products of GO-spaces [4]. In this paper, we characterize the hereditary paracompactness of lexicographic products of GO-spaces and get some applications. For example, we see:

- the lexicographic products $\mathbb{S}^\gamma$, $\mathbb{M}^\gamma$, $\mathbb{R}^\gamma$ and $(0,1)_{\mathbb{R}}^\gamma$ are hereditarily paracompact for every ordinal γ, where $\mathbb{S}$ and $\mathbb{M}$ denote the Sorgenfrey line and Michael line respectively,
- the lexicographic product $[0,1)_{\mathbb{R}}^\omega$ is hereditarily paracompact, but the lexicographic product $[0,1)_{\mathbb{R}}^{\omega_1}$ is not hereditarily paracompact,
- the lexicographic product $\omega_1 \times (0,1]_{\mathbb{R}}$ is hereditarily paracompact but the lexicographic product $\omega_1 \times [0,1)_{\mathbb{R}}$ is not paracompact,
- the lexicographic product $(\omega_1^2 \times (-\omega_1)^3)^{\omega_1}$ is hereditarily paracompact, but the lexicographic products ω_1^ω and $\omega_1^{\omega_1}$ are not paracompact, where for a GO-space $X = \langle X, <_X, \tau_X\rangle$, $-X$ denotes the GO-space $\langle X, >_X, \tau_X\rangle$.

2010 *Mathematics Subject Classification.* Primary 54F05, 54B10, 54B05 . Secondary 54C05.

Key words and phrases. Lexicographic product, GO-space, LOTS, paracompact, hereditarily paracompact.