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HOMOTOPY GROUPS OF INFINITE WEDGE

LEONARD MDZINARISHVILI

Abstract. In *Homotopy Theory* (Pure and Applied Mathematics, Vol. VIII, Academic Press, New York–London, 1959), Sze-tsen Hu proved for $X \vee Y$, the wedge sum of pointed spaces (X, x_0), and (Y, y_0) that for $n \geq 2$ there is an isomorphism

(1) $\pi_n(X \vee Y, u_0) \approx \pi_n(X, x_0) \oplus \pi_n(Y, y_0) \oplus \pi_{n+1}(X \times Y, X \vee Y, u_0)$,

where $u_0 = (x_0, y_0)$.

This result was not generalized for an infinite wedge $\vee Y_\omega$, $\omega \in \Omega$, of pointed spaces (Y_ω, y_ω^0) in view of the fact that an infinite wedge $\vee Y_\omega$ is not a subspace of the direct product $\prod Y_\omega$, $\omega \in \Omega$.

In the present work we prove that for $n \geq 2$ there is an isomorphism

$$\pi_n(\vee Y_\omega, y^0) \approx \sum_{\omega \in \Omega} \pi_n(Y_\omega, y_\omega^0) \oplus \pi_{n+1}(LY_\omega, \vee Y_\omega, y^0),$$

where LY_ω is the weak product of pointed topological spaces (Y_ω, y_ω^0), $\omega \in \Omega$ (see C. J. Knight, *Weak products of spaces and complexes*, Fund. Math. **53** (1963), 1–12.)

Let Top_* be the category of pointed topological spaces and continuous maps preserving base point [4].

If $\Omega = \{\omega\}$ is an infinite set and $\{(Y_\omega, y_\omega^0)\}_{\omega \in \Omega}$ is a family of objects from Top_* indexed by Ω, their infinite wedge is denoted by $\vee Y_\omega$ and is defined by $\bigcup\limits_{\omega \in \Omega} Y_\omega / \bigcup y_\omega^0$ the quotient space of $\bigcup Y_\omega$ obtained by identifying all of $\bigcup y_\omega^0$ to a single point u^0. We define a topology by declaring a subset $U \subset \bigcup\limits_{\omega \in \Omega} Y_\omega$ to be open if and only if the intersection $U \cap Y_\omega$ is open in Y_ω for all $\omega \in \Omega$ [1, Definition 2.2.8].

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