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A FAMILY OF GENERALIZED INVERSE LIMITS HOMEOMORPHIC TO "THE MONSTER"

FARUQ MENA AND ROBERT P. ROE

Abstract. We show that two generalized inverse limit spaces that one might suspect are not homeomorphic are in fact homeomorphic.

1. Introduction and Definitions

We are interested in the family of upper semi-continuous functions $f_a : [0,1] \to [0,1]$ and the corresponding inverse limits $X_a = \varprojlim\{[0,1], f_a\}$, where the graph $\gamma(f_a)$ of f_a is the union of the line segments from $(0,0)$ to $(a,1)$ to $(1,a)$ to $(1,0)$ for $a \in [0,1]$. For $a \in (0,1)$, f_a is a generalized upper semi-continuous (usc) Markov function and it follows from results of Iztok Banič and Tjaša Lunder [1] that if $a, b \in (0,1)$, then X_a is homeomorphic to X_b. But for $a \in (0,1)$, X_a and X_1 are not homeomorphic since the first contains the topologist's sine curve as a subcontinuum and the second is the harmonic fan. The functions f_a where $a \neq 0$, and f_0 do not satisfy the hypothesis of Banić and Lunder's theorem so we may ask, are $X_{\frac{1}{2}}$ and X_0 homeomorphic? In his master's thesis, Christopher David Jacobsen [4] studies $X_{\frac{1}{2}}$ where he shows that it contains $2^{\aleph_0}$ arc components and each arc component is dense. The space X_0 is often referred to as "the monster," a name reportedly coined by Banić.

Several other authors also have results showing when families of functions have homeomorphic inverse limits. For example, W. T. Ingram and William S. Mahavier [3] have shown that if f and g are usc functions which are topologically conjugate, then the corresponding inverse limit spaces are homeomorphic. Michel Smith and Scott Varagona [6] have shown that

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