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SPLITTING OF COLLAPSING MAPS FOR FREE ABELIAN TOPOLOGICAL GROUPS

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Abstract. We prove the following.

Theorem. *Suppose that X is a finite complex and Y is a connected subcomplex such that $H^{i+1}(X/Y; H_i(Y)) = 0$ for all $i > 0$. Then, for free abelian topological groups,*

$$\mathbb{A}(X) \cong \mathbb{A}(X/Y) \times \mathbb{A}(Y).$$

As a corollary, we obtain that $\mathbb{A}(\mathbb{C}P^2) = \mathbb{A}(S^2 \vee S^4)$, whereas $\mathbb{F}(\mathbb{C}P^2) \neq \mathbb{F}(S^2 \vee S^4)$ where $\mathbb{F}(X)$ denotes free topological group generated by X.

1. Introduction

The free topological group $\mathbb{F}(X)$ and the free abelian topological group $\mathbb{A}(X)$ generated by a topological space X were defined first by A. A. Markov [7] and [8] in the topological category and then by M. I. Graev [6] in the pointed topological category. Markov's and Graev's definitions are closely related. In this paper, we consider the latter. We consider these groups for finite CW complexes X. Note that there are natural embeddings $X \subset \mathbb{F}(X)$ and $X \subset \mathbb{A}(X)$ where the base point x_0 is identified with the unit.

The defining property of $\mathbb{A}(X)$ is the following: *For every continuous map $f : X \to G$ of a pointed compact metric space (X, e) to a topological abelian group G with $f(e) = 0 \in G$, there is a unique extension to a continuous homomorphism $\bar{f} : \mathbb{A}(X) \to G$.* For $\mathbb{F}(X)$, the defining property is similar (see [2]).

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