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ENDPOINTS OF INVERSE LIMITS FOR A FAMILY OF SET-VALUED FUNCTIONS

LORI ALVIN AND JAMES P. KELLY

Abstract. We study the endpoints of inverse limits of set-valued functions. In a previous article (2016), one of the authors studied this topic using R. H. Bing's definition of endpoints (most often associated with chainable continua), and showed that if a set-valued function F has its inverse equal to the union of continuous, single-valued functions, then a point $\mathbf{p} = (p_0, p_1, \ldots)$ is an endpoint of $\varprojlim F$ if and only if $\pi_{[0,n]}(\mathbf{p})$ is an endpoint of $\pi_{[0,n]}(\varprojlim F)$ for infinitely many $n \in \mathbb{N}$. The question was posed whether this same result would hold if instead we used A. Lelek's definition of endpoint (most often associated with dendroids).

We present an example giving a negative answer to this question. We go on to give characterizations for the sets of endpoints for a family of set-valued functions. These functions have graphs which consist of a symmetric tent map and a straight line connecting the critical point to either $(0,1)$, $(\frac{1}{2},1)$, or $(1,1)$. The endpoints of inverse limits of tent maps are well-studied, but we show that the addition of the straight line fundamentally alters the set of endpoints.

1. Introduction

Suppose that $F : [0,1] \to 2^{[0,1]}$ is an upper semi-continuous set-valued function and that $\mathbf{p} \in \varprojlim F$. Assume the following definition of an endpoint of a continuum: p is an *endpoint* of the continuum X if for any two subcontinua $H, K \subseteq X$ which both contain p, either $H \subseteq K$ or $K \subseteq H$. (This definition is given by R. H. Bing in [4] and is primarily used in the context of arc-like continua.) Using this definition, it is shown in [7, Theorem 1.2] that $\mathbf{p}$ is an endpoint of $\varprojlim F$ provided that for infinitely many

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