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by

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Electronically published on April 11, 2019

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**Topology Proceedings** 

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124
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E-Published on April 11, 2019

## MAHAVIER PRODUCTS, IDEMPOTENT RELATIONS, AND CONDITION $\boldsymbol{\Gamma}$

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ABSTRACT. Clearly, a generalized inverse limit of metrizable spaces indexed by N is metrizable, as it is a subspace of a countable product of metrizable spaces. The authors previously showed that all idempotent, upper semi-continuous, surjective, continuum-valued bonding functions on [0, 1] (other than the identity) satisfy a certain Condition  $\Gamma$ ; it follows that only in trivial cases can a generalized inverse limit of copies of [0, 1] indexed by an uncountable ordinal be metrizable. The authors show that Condition  $\Gamma$  is, in fact, guaranteed by much weaker criteria, proving a more general metrizability theorem for certain Mahavier products.

## 1. INTRODUCTION

In the spirit of the celebrated work by William S. Mahavier [9] and by W. T. Ingram and Mahavier [7], who first generalized traditional inverse limits to those with set-valued bonding functions, researchers have sought more ways to generalize inverse limits. One route proposed in [8] would be to index the factor spaces of an inverse limit not necessarily by the natural numbers, but rather by some other directed set. However, a poorly-behaved directed set can cause an inverse limit to be empty [8]; therefore, most research in this area has involved inverse limits with totally ordered index sets.

Various results in recent years have shown that investigating totally ordered index sets other than the natural numbers is fertile ground for new work. Ingram and Mahavier lay the foundation in [8] by proving some

<sup>2010</sup> Mathematics Subject Classification. 54B10 54C60 54D80 54F15.

Key words and phrases. idempotence, Mahavier product, relations, set valued, upper semi continuous.

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