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by

Boris Okun and Richard Scott

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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GROWTH SERIES OF CAT(0) CUBICAL COMPLEXES

BORIS OKUN AND RICHARD SCOTT

ABSTRACT. Let X be a CAT(0) cubical complex. The growth series of X at x is $G_x(t) = \sum_{y \in Vert(X)} t^{d(x,y)}$, where $d(x,y)$ denotes ℓ_1-distance between x and y. If X is cocompact, then G_x is a rational function of t. In the case when X is the Davis complex of a right-angled Coxeter group it is a well known that $G_x(t) = 1/f_L(-t/(1+t))$, where f_L denotes the f-polynomial of the link L of a vertex of X. We obtain a similar formula for general cocompact X. We also obtain a simple relation between the growth series of individual orbits and the f-polynomials of various links. In particular, we get a simple proof of reciprocity of these series ($G_x(t) = \pm G_x(t^{-1})$) for an Eulerian manifold X.

Let X be a CAT(0) cube complex with a cocompact cellular action by a group G. Denote by $d(x,y)$ the ℓ_1-distance between vertices x and y of X. We consider the following growth series:

$$G_{xy} = \sum_{z \in Gy} t^{d(x,z)}$$

— the growth series of G-orbit of y as seen from x, and

$$G_x = \sum_{y \in X} t^{d(x,y)}$$

— the full growth series of X as seen from x.

The aim of this paper is to establish relations between these growth series and the local structure of X and X/G. In order to do this we

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