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## ON THE NOTION OF TREE-LIKENESS FOR GENERALIZED CONTINUA

## WŁODZIMIERZ J. CHARATONIK, TOMÁS FERNÁNDEZ-BAYORT, AND ANTONIO QUINTERO

ABSTRACT. A variety of equivalent approaches to tree-likeness is available in classical continuum theory. In absence of compactness, some of those equivalences do not hold. In this paper, we compare the class of generalized continua defined as inverse limits of locally finite trees with proper bonding maps with the class of those for which any open cover admits acyclic refinements. We show that the latter is precisely the subclass of the former consisting of those generalized continua with exhausting sequences of tree-like continua. In addition, we show that locally injective proper maps onto tree-like generalized continua are homeomorphisms for the second definition but not for the first one, which, notwithstanding, is still reflected by such maps.

## 1. INTRODUCTION

The proper category is widely accepted as the most convenient framework for the study of the topology of locally compact spaces; in particular, classes of spaces and maps of interest in continuum theory are extended to the proper category. Recall that a map  $f: X \to Y$  is said to be *proper* (also termed *perfect* in the literature) if for any compact subset  $K \subset Y$ ,  $f^{-1}(K)$  is compact in X. It is well known that proper maps are closed maps [6, Theorem 3.7.18].

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