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A BANACH-STONE TYPE THEOREM AND TOPOLOGICAL DIMENSION

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Abstract. We prove a Banach-Stone type theorem for linear isometries of vector-valued continuous function spaces in connection with topological dimensions of underlying spaces, generalizing a previous result of the author. Some related problems are discussed.

1. Introduction and Preliminaries

This paper is a continuation of [9] and studies linear isometries between vector-valued continuous function spaces. The classical Banach-Stone theorem states that every surjective linear isometry between the Banach spaces of all complex-valued continuous functions on compact Hausdorff spaces is a unimodular weighted composition operator (see [5], [6] and [8] for background). Seeking for analogous theorems for isometries between vector-valued continuous function spaces and being motivated by [1], [2], [7] and [15], we ask the following question. For a compact Hausdorff space X and a Banach space E, $C(X,E)$ denotes the Banach space of all E-valued continuous functions on X with the sup norm $\|f\|_\infty = \sup_{x\in X}\|f(x)\|$. Undefined notation will be explained later.

Question 1.1. Let X and Y be compact Hausdorff spaces, let E be a real or complex Banach space and let A and B be linear subspaces of $C(X,E)$ and $C(Y,E)$ respectively. Find a set of conditions on X, Y, A, B and E which implies that every surjective linear isometry $T : A \to B$

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