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PRODUCTS OF REGULAR LOCALLY COMPACT SPACES ARE $K_{\mathbb{R}}$-SPACES

HELGE GLÖCKNER AND NIKU MASBOUGH

Abstract. A theorem by N. Noble from 1970 asserts that every cartesian product of completely regular, locally pseudo-compact $k_{\mathbb{R}}$-spaces is a $k_{\mathbb{R}}$-space. As a consequence, all products of locally compact Hausdorff spaces are $k_{\mathbb{R}}$-spaces. We provide an accessible proof for this fact. More generally, we show that all products of not necessarily Hausdorff, regular locally compact spaces are $k_{\mathbb{R}}$-spaces.

1. Introduction

Recall that a (not necessarily Hausdorff) topological space X is called *compact* if every open cover of X has a finite subcover, and *locally compact* if every neighborhood of a point in X contains a compact neighborhood. If every neighborhood contains a closed neighborhood, then X is called *regular*; it is *completely regular* if the topology on X is initial with respect to the set $C(X,\mathbb{R})$ of all continuous real-valued functions on X. A function $f\colon X\to Y$ to a topological space Y is called *k-continuous* if its restriction $f|_K\colon K\to Y$ is continuous for each compact subset $K\subseteq X$. If every k-continuous real-valued function on X is continuous, then X is called a *$k_{\mathbb{R}}$-space.* Every k-continuous function from a $k_{\mathbb{R}}$-space to a completely regular topological space is continuous. We shall always endow cartesian products of topological spaces with the product topology. Stimulated by [4], we prove the following theorem:

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Key words and phrases. Compactly generated space, cartesian product, product topology, locally compact space, regular space, Hausdorff property.

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