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by

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THE POLISH TOPOLOGY OF ERDŐS SPACE

JAN VAN MILL

Abstract. We show that Erdős space E is Polishable and prove that E with its Polish topology is homeomorphic to complete Erdős space.

1. Introduction

A topological group is *Polishable* if it admits a stronger Polish topology that is compatible with its group structure. If a topological group is Polishable then it is obviously Borel. But this is not sufficient, see Becker and Kechris [1, p. 12]. If a topological group is Polishable, then its Polish topology is unique; see Kechris [8, Theorem 9.10]. Hence the property of being Polishable is an intrinsic property of the topological group we are interested in. The reader can find more information on Polishable groups for example in Solecki [11].

As usual, $\mathbb{Q}$ and $\mathbb{P}$ denote the sets of rationals and irrationals, respectively.

The aim of this note is to show that Erdős space

$$E = \{x \in \ell^2 : (\forall\, n \in \mathbb{N})(x_n \in \mathbb{Q})\}$$

from [6] is Polishable. We will show that E with its Polish topology is homeomorphic to *complete* Erdős space

$$E_c = \{x \in \ell^2 : (\forall\, n \in \mathbb{N})(x_n \in (\{0\} \cup \{{}^1\!/_n : n \in \mathbb{N}\})\},$$

which was also considered in [6]. It is known, see [7] (and [2, 3]), that this space is homeomorphic to $\{x \in \ell^2 : (\forall\, n \in \mathbb{N})(x_n \in \mathbb{P})\}$.

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