http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 55, 2020

Pages 87–114

http://topology.nipissingu.ca/tp/

An Extension of the Baire Property

by

Christopher Caruvana and Robert R. Kallman

Electronically published on May 11, 2019

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 55 (2020)
Pages 87-114

E-Published on May 11, 2019

AN EXTENSION OF THE BAIRE PROPERTY

CHRISTOPHER CARUVANA AND ROBERT R. KALLMAN

Abstract. The purpose of this paper is to define for every Polish space X a class of sets, the $EBP(X)$-sets or the extended Baire property sets, to work out many properties of the $EBP(X)$-sets and to show their usefulness in analysis. For example, a proper generalization of the Pettis Theorem is proved in this context that furnishes a new automatic continuity result for Polish groups. The name extended Baire property sets is reasonable since $EBP(X)$ contains the Baire property sets $BP(X)$ and it is consistent with ZFC that the containment is proper.

1. Introduction

The purpose of this paper is to define for every Polish space X a class of sets, the $EBP(X)$ sets or the extended Baire property sets, to work out many properties of the $EBP(X)$ sets and to show their usefulness in analysis. For example, a proper generalization of the Pettis Theorem is proved in this context that furnishes a new automatic continuity result for Polish groups. The name extended Baire property sets is reasonable since $EBP(X)$ contains the Baire property sets $BP(X)$ and it is consistent with ZFC that the containment is proper.

Recall some basic facts about the topology on the space of probability measures on a Polish space X. Let $\mathcal{M}(X)$ be the collection of all Borel probability measures on X and let $C_b(X)$ be the collection of all functions $f : X \to \mathbb{R}$ which are continuous and bounded. Endow $\mathcal{M}(X)$ with the coarsest topology for which each map

$$\mu \mapsto \int f \ d\mu, \mathcal{M}(X) \to \mathbb{R},$$

2010 *Mathematics Subject Classification.* 03E15, 03E35, 54H11, 28A05, 28C15, 28C10.

Key words and phrases. Measure, Category, Polish Groups.