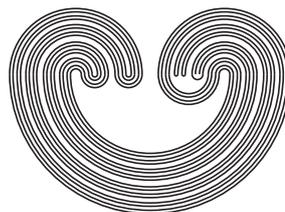


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## SPACES WITH NO S OR L SUBSPACES

by

JOAN E. HART AND KENNETH KUNEN

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## SPACES WITH NO S OR L SUBSPACES

JOAN E. HART AND KENNETH KUNEN

**ABSTRACT.** We consider spaces that contain neither an S-space nor an L-space. We call such a space ESLC, and show that it is consistent with ZFC for a product of two ESLC spaces to contain both an S-space and an L-space.

### 1. INTRODUCTION

All topological spaces considered in this paper are  $T_3$  (Hausdorff and regular).

The following terminology is standard: A space  $X$  is *hereditarily separable* (HS) iff all subspaces of  $X$  are separable, and *hereditarily Lindelöf* (HL) iff all subspaces of  $X$  are Lindelöf. Then,  $X$  is an *S-space* iff  $X$  is HS but not HL, and  $X$  is an *L-space* iff  $X$  is HL but not HS.

S-spaces are consistent with  $\text{MA}(\aleph_1)$  [14], but are refuted by PFA [16], while L-spaces exist in ZFC [10]. Under CH, a large variety of S-spaces and L-spaces have been constructed, and still more have been built under  $\diamond$ .

In this paper, we shall study the following notion:

**Definition 1.1.** The space  $X$  is *ESLC* iff every subspace of  $X$  is either both HS and HL or neither HS nor HL.

This is the same as saying that no subspace of  $X$  is either an S-space or an L-space. The reason for the “C” in “ESLC” is the following slightly non-standard terminology for a standard concept:

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