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CONNECTEDNESS OF THE FINE TOPOLOGY

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Abstract. This paper studies the connectedness of the fine topology on $C(X,Y)$, the set of all continuous functions from a Tychonoff space X to a metric space (Y,d). Occasionally, we assume Y to be a normed linear space. We also determine the components of the space $H(\mathbb{R}^n)$, of all self homeomorphisms on the n-dimensional Euclidean space $\mathbb{R}^n$, where $H(\mathbb{R}^n)$ is considered as a subspace of the space $C(\mathbb{R}^n,\mathbb{R}^n)$ equipped with the fine topology.

1. Introduction

The set $C(X,Y)$ of all continuous functions from a topological space X into a metric space Y has a number of natural topologies, such as the point-open, compact-open, and uniform topologies (see, for example, [1], [2], [7], [17], [26] and [28]). When $Y=\mathbb{R}$, the space of real numbers, we write $C(X)$ instead of $C(X,\mathbb{R})$. Although the point-open and compact-open topologies on $C(X,Y)$ can be defined for any space Y, in order to define the uniform topology, it is necessary for Y to have some uniform structure. The *uniform topology on* $C(X,Y)$ *for a metric space* (Y,d) has basic open sets of the form

$$B_d(f,\delta)=\{h\in C(X,Y):\sup\{d(h(x),f(x)):x\in X\}<\delta\},$$

where $f\in C(X,Y)$ and δ is a positive constant. Let $C_d(X,Y)$ denote the space $C(X,Y)$ equipped with the uniform topology generated by the metric d on Y. However, sometimes none of the above topologies is strong enough to apply a function space to a given situation, in which case a finer topology may be needed so that we have a stronger convergence.

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