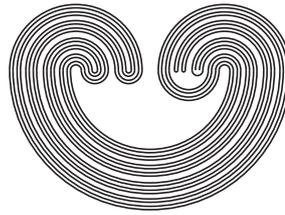


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## CONNECTEDNESS OF THE FINE TOPOLOGY

by

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## CONNECTEDNESS OF THE FINE TOPOLOGY

VARUN JINDAL AND ANUBHA JINDAL

**ABSTRACT.** This paper studies the connectedness of the fine topology on  $C(X, Y)$ , the set of all continuous functions from a Tychonoff space  $X$  to a metric space  $(Y, d)$ . Occasionally, we assume  $Y$  to be a normed linear space. We also determine the components of the space  $H(\mathbb{R}^n)$ , of all self homeomorphisms on the  $n$ -dimensional Euclidean space  $\mathbb{R}^n$ , where  $H(\mathbb{R}^n)$  is considered as a subspace of the space  $C(\mathbb{R}^n, \mathbb{R}^n)$  equipped with the fine topology.

### 1. INTRODUCTION

The set  $C(X, Y)$  of all continuous functions from a topological space  $X$  into a metric space  $Y$  has a number of natural topologies, such as the point-open, compact-open, and uniform topologies (see, for example, [1], [2], [7], [17], [26] and [28]). When  $Y = \mathbb{R}$ , the space of real numbers, we write  $C(X)$  instead of  $C(X, \mathbb{R})$ . Although the point-open and compact-open topologies on  $C(X, Y)$  can be defined for any space  $Y$ , in order to define the uniform topology, it is necessary for  $Y$  to have some uniform structure. The *uniform topology on  $C(X, Y)$  for a metric space  $(Y, d)$*  has basic open sets of the form

$$B_d(f, \delta) = \{h \in C(X, Y) : \sup\{d(h(x), f(x)) : x \in X\} < \delta\},$$

where  $f \in C(X, Y)$  and  $\delta$  is a positive constant. Let  $C_d(X, Y)$  denote the space  $C(X, Y)$  equipped with the uniform topology generated by the metric  $d$  on  $Y$ . However, sometimes none of the above topologies is strong enough to apply a function space to a given situation, in which case a finer topology may be needed so that we have a stronger convergence.

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*Key words and phrases.* Spaces of continuous functions, uniform topology, fine topology, quasicomponent, connectedness, locally connected, totally disconnected, spaces of homeomorphisms.

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