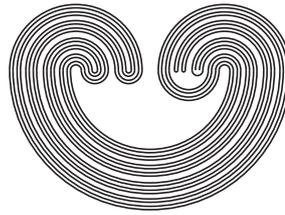


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 55, 2020

Pages 265–271

---

<http://topology.nipissingu.ca/tp/>

## TOPOLOGICAL CONJUGACY FOR THE MORSE MINIMAL SYSTEM: AN EXAMPLE

by

ANDREW DYKSTRA AND MICHELLE LEMASURIER

Electronically published on February 3, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## TOPOLOGICAL CONJUGACY FOR THE MORSE MINIMAL SYSTEM: AN EXAMPLE

ANDREW DYKSTRA AND MICHELLE LEMASURIER

**ABSTRACT.** We give an example of a subshift that is topologically conjugate to the Morse minimal system but is not generated by a constant-length substitution. This shows that the property of being generated by a constant-length substitution is not a topological conjugacy invariant. Our proof illustrates the usefulness of techniques developed by Coven, Dekking, and Keane [3].

### 1. INTRODUCTION AND PRELIMINARIES

Given a finite alphabet  $\mathcal{A}$ , let  $\mathcal{A}^{\mathbb{Z}}$  denote the set of doubly-infinite sequences  $x = (x_i)_{i \in \mathbb{Z}}$  on  $\mathcal{A}$ , and let  $\sigma : \mathcal{A}^{\mathbb{Z}} \rightarrow \mathcal{A}^{\mathbb{Z}}$  be the left shift:  $\sigma(x)_i = x_{i+1}$ . If a subset  $X \subseteq \mathcal{A}^{\mathbb{Z}}$  is  $\sigma$ -invariant and closed in the product topology, then the pair  $(X, \sigma)$  is a *subshift*. The subshift is *minimal* if  $X = \overline{\{\sigma^n(x) : n \in \mathbb{Z}\}}$  for every  $x \in X$ . Given another subshift  $(Y, \sigma)$ , a continuous function  $\psi : X \rightarrow Y$  such that  $\psi \circ \sigma = \sigma \circ \psi$  is called a *factor map* if it is surjective, and a *topological conjugacy* if it is a homeomorphism. For more background on subshifts, see [10].

Given an integer  $L \geq 2$ , a *substitution* of constant length  $L$  is a mapping  $\theta : \mathcal{A} \rightarrow \mathcal{A}^L$ . Higher powers  $\theta^k : \mathcal{A} \rightarrow \mathcal{A}^{L^k}$  are defined recursively: if  $\theta(a) = a_1 \cdots a_\ell$ , then  $\theta^2(a) := \theta(a_1) \cdots \theta(a_\ell)$ ;  $\theta^3(a) := \theta^2(a_1) \cdots \theta^2(a_\ell)$ ; and so on. The substitution is *primitive* if, for all  $a, b \in \mathcal{A}$ , there exists  $k \in \mathbb{N}$  such that  $\theta^k(a)$  contains  $b$ . It is well-known that a primitive

---

2010 *Mathematics Subject Classification.* Primary 37B10; Secondary 54H20.

*Key words and phrases.* Symbolic dynamics, substitutions, topological conjugacy, Morse minimal system.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.