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Carl Hammarsten, Rae Helmreich, Anchala Krishnan, John Meier, and Nathan Schmitz

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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PERSISTENT HOMOLOGY AND RANDOM MODELS OF THE GAUSSIAN PRIMES

CARL HAMMARSTEN, RAE HELMREICH, ANCHALA KRISHNAN, JOHN MEIER, AND NATHAN SCHMITZ

Abstract. Ilan Vardi introduced a probabilistic model for the distribution of Gaussian primes in 1998 (see *Prime percolation*, Experiment. Math. **7** (1998), no. 3, 275–289). We use persistent homology to test how well Vardi's model and a symmetrized variant of Vardi's model capture the geometry of the Gaussian primes. An analysis of the persistent first homology of point clouds produced by these models provides statistical evidence that the models miss some fundamental geometric features of the Gaussian primes.

1. Introduction

The Gaussian primes are the irreducible elements in the Gaussian integers (the ring $\mathbb{Z}[i]$). There are elementary, open questions about the geometry of the Gaussian primes thought of as a point cloud in $\mathbb{R}^2$. In 1962 at the International Congress of Mathematicians in Stockholm, Basil Gordon asked if you can "walk to infinity" stepping only on Gaussian primes. Said a bit more formally, for any positive real number D, define $\mathcal{G}_D$ to be the graph whose vertices consist of Gaussian primes and whose edges are the pairs of primes $\{p, q\}$ where $d(p, q) \leq D$. (The case when

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