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TOPOLOGY PROCEEDINGS

Volume 56, 2020

Pages 57–70

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Electronically published on August 1, 2019

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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THE HYPERSPACE OF SEQUENCES OF A DENDRITE IS CONTRACTIBLE

ALEJANDRO ILLANES

Abstract. Given a Hausdorff space X, let $S_c(X)$ be the hyperspace of nontrivial convergent sequences. It is an open problem to determine if the contractibility of X implies the contractibility of $S_c(X)$. In this paper we prove that if X is a dendrite (a locally connected continuum without simple closed curves), then $S_c(X)$ is contractible. This answers a question by Javier Camargo, David Maya, and Patricia Pellicer-Covarrubias [*Path connectedness, local path connectedness and contractibility of* $S_c(X)$. Available at arXiv:1802.00725v1 [math.GN].]

1. Introduction

The *harmonic sequence* is the subspace $\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\}$ of the real line. Given a Hausdorff space X, the *hyperspace of nontrivial convergent sequences* $S_c(X)$ is defined as

$S_c(X) = \{A \subset X : A$ is homeomorphic to the harmonic sequence$\}$.

This hyperspace is considered with the Vietoris topology. In the case that X is a metric space, the Vietoris topology in $S_c(X)$ is induced by the Hausdorff metric H.

The hyperspace $S_c(X)$ is introduced by S. García-Ferreira and Y. F. Ortiz-Castillo in [3]. Several authors study different aspects related to these hyperspaces in [2]–[10]. A very natural problem is the following.

2010 *Mathematics Subject Classification.* Primary 54B20; Secondary 54F15, 54F55.

Key words and phrases. continuum, contractibility, dendrite, harmonic fan, hyperspace, hyperspace of nontrivial convergent sequences.

This paper was partially supported by the projects "Teoría de Continuos, Hiperespacios y Sistemas Dinámicos III" (IN106319) of PAPIIT, DGAPA, UNAM; and "Teoría de Continuos e Hiperespacios" (A1-S-15492) of CONACyT.