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by

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UNDECIDABILITY OF THE CARDINALITY OF C^*-EMBEDDED DISCRETE SUBSETS IN PRODUCTS OF NATURAL NUMBERS

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Abstract. Let $\mathbb{N}$ denote $\{1, 2, \cdots\}$ with the discrete topology. In this paper, we prove that it is undecidable in ZFC that every C^*-embedded (or C-embedded) discrete subset in $\mathbb{N}^{\mathfrak{c}}$ is countable, where $\mathfrak{c} = 2^\omega$. We also show that there is a C^*-embedded discrete subset in $\mathbb{N}^\kappa$ with cardinality κ only if κ has countable cofinality.

1. Introduction

For a set A, we denote by $|A|$ the cardinality of A. All cardinals dealt with here (κ and τ, etc.) are assumed to be infinite. In particular, ω, ω_1, and $\mathfrak{c}$ denote the first infinite cardinal, the first uncountable cardinal, and the cardinal of continuum, respectively.

We denote by CH the continuum hypothesis (i.e., $\mathfrak{c} = \omega_1$). Let $\mathbb{N}$ denote $\{1, 2, \cdots\}$ with the discrete topology. For a space X, we denote by X^κ the product of κ many copies of X. In particular, we deal with $\mathbb{N}^\kappa$. Let $w(X)$ denote the weight of X. For a Tychonoff space X, we denote by βX the Čech-Stone compactification of X.

For a space X, $A \subset X$ is *C^*-embedded* (*C-embedded*) in X if every continuous function from A to the unit interval $\mathbb{I} = [0, 1]$ (the real-line $\mathbb{R}$) can be continuously extended over X.

In [10], Elżbieta Pol and Roman Pol prove the following result.

(1) There is a closed discrete countable subset in $\mathbb{N}^{\mathfrak{c}}$ which is C^*-embedded but not C-embedded in $\mathbb{N}^{\mathfrak{c}}$.

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