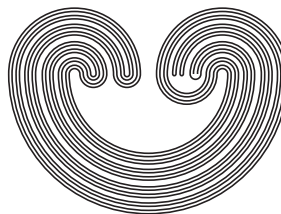


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## UNDECIDABILITY OF THE CARDINALITY OF $C^*$ -EMBEDDED DISCRETE SUBSETS IN PRODUCTS OF NATURAL NUMBERS

by

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**UNDECIDABILITY OF THE CARDINALITY OF  
 $C^*$ -EMBEDDED DISCRETE SUBSETS IN  
PRODUCTS OF NATURAL NUMBERS**

YASUSHI HIRATA AND YUKINOBU YAJIMA

**ABSTRACT.** Let  $\mathbb{N}$  denote  $\{1, 2, \dots\}$  with the discrete topology. In this paper, we prove that it is undecidable in **ZFC** that every  $C^*$ -embedded (or  $C$ -embedded) discrete subset in  $\mathbb{N}^c$  is countable, where  $c = 2^\omega$ . We also show that there is a  $C^*$ -embedded discrete subset in  $\mathbb{N}^\kappa$  with cardinality  $\kappa$  only if  $\kappa$  has countable cofinality.

**1. INTRODUCTION**

For a set  $A$ , we denote by  $|A|$  the cardinality of  $A$ . All cardinals dealt with here ( $\kappa$  and  $\tau$ , etc.) are assumed to be infinite. In particular,  $\omega$ ,  $\omega_1$ , and  $\mathfrak{c}$  denote the first infinite cardinal, the first uncountable cardinal, and the cardinal of continuum, respectively.

We denote by **CH** the continuum hypothesis (i.e.,  $\mathfrak{c} = \omega_1$ ). Let  $\mathbb{N}$  denote  $\{1, 2, \dots\}$  with the discrete topology. For a space  $X$ , we denote by  $X^\kappa$  the product of  $\kappa$  many copies of  $X$ . In particular, we deal with  $\mathbb{N}^\kappa$ . Let  $w(X)$  denote the weight of  $X$ . For a Tychonoff space  $X$ , we denote by  $\beta X$  the Čech-Stone compactification of  $X$ .

For a space  $X$ ,  $A \subset X$  is  $C^*$ -embedded ( $C$ -embedded) in  $X$  if every continuous function from  $A$  to the unit interval  $\mathbb{I} = [0, 1]$  (the real-line  $\mathbb{R}$ ) can be continuously extended over  $X$ .

In [10], Elżbieta Pol and Roman Pol prove the following result.

(1) There is a closed discrete countable subset in  $\mathbb{N}^c$  which is  $C^*$ -embedded but not  $C$ -embedded in  $\mathbb{N}^c$ .

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