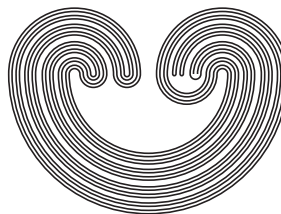


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## COUNTABLE DENSE HOMOGENEITY OF FUNCTION SPACES

by

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## COUNTABLE DENSE HOMOGENEITY OF FUNCTION SPACES

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**ABSTRACT.** In this paper, we consider the question of when the space  $C_p(X)$  of continuous real-valued functions on  $X$  with the pointwise convergence topology is countable dense homogeneous. In particular, we focus on the case when  $X$  is countable with a unique non-isolated point  $\infty$ . In this case,  $C_p(X)$  is countable dense homogeneous if and only if the filter of open neighborhoods of  $\infty$  is a non-meager  $P$ -filter.

### 1. INTRODUCTION

All spaces considered are assumed to be Tychonoff.

A space  $X$  is countable dense homogeneous (CDH, henceforth) if  $X$  is separable and, whenever  $D, E \subset X$  are countable dense subsets, there is a homeomorphism  $h: X \rightarrow X$  such that  $h[D] = E$ . Among examples of CDH spaces we have the Euclidean spaces, the Hilbert cube, and the Cantor set. For updated surveys on CDH spaces, see [2, §§14, 15, 16] and [10].

One of the most notable open problems in the theory of CDH spaces is the existence of metric CDH spaces that are not Polish; this is [6, Problem 6]. This problem is solved in [5] where the authors construct a meager-in-itself CDH subspace of the reals. Later, another example of a CDH non-Polish subspace of the reals that is a Baire space is given in [9].

Another related result is that  $\mathbb{R}^\kappa$  is CDH if and only if  $\kappa < \mathfrak{p}$ . This result is proved in two steps: first, J. Steprāns and H. X. Zhou [20] prove

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*Key words and phrases.* countable dense homogeneous, non-meager  $P$ -filter, pointwise convergence topology.

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