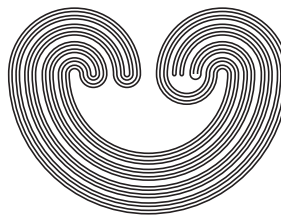


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# TOPOLOGY PROCEEDINGS



Volume 56, 2020

Pages 147–159

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<http://topology.nipissingu.ca/tp/>

## CELLULAR COMPACTNESS IN FUNCTION SPACES

by

VLADIMIR V. TKACHUK

Electronically published on October 21, 2019

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**ISSN:** (Online) 2331-1290, (Print) 0146-4124

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## CELLULAR COMPACTNESS IN FUNCTION SPACES

VLADIMIR V. TKACHUK

**ABSTRACT.** Given a Tychonoff space  $X$ , we provide some necessary and sufficient conditions (in terms of the topology of the space  $X$ ) for  $C_p(X, [0, 1])$  to be cellular-compact. We show that countable compactness of  $C_p(X, [0, 1])$  implies its cellular compactness, but cellular compactness of  $C_p(X, [0, 1])$  does not imply existence of a dense countably compact subspace in the space  $C_p(X, [0, 1])$ . We also establish that  $C_p(X)$  is  $\sigma$ -cellular-compact if and only if  $X$  is finite. Besides, pseudocompleteness of  $C_p(X)$  implies that  $C_p(X, [0, 1])$  is cellular-compact.

### 1. INTRODUCTION

Angelo Bella and Santi Spadaro introduce in [1] the class of cellular-Lindelöf spaces. Recall that a space  $X$  is *cellular-Lindelöf* if for any disjoint family  $\mathcal{U}$  of non-empty open subsets of  $X$ , there exists a Lindelöf subspace  $L \subset X$  such that  $L \cap U \neq \emptyset$  for any  $U \in \mathcal{U}$ . Wei-Feng Xuan and Yan-Kui Song construct in [12] an example of a weakly Lindelöf space that is not cellular-Lindelöf. V. V. Tkachuk proves in [10] that cellular-Lindelöf spaces need not be weakly Lindelöf, while Bella and Spadaro establish in [2] that every cellular-Lindelöf monotonically normal space is Lindelöf and prove, under  $2^{<c} = \mathfrak{c}$ , that every normal cellular-Lindelöf first countable space has cardinality not greater than  $\mathfrak{c}$ .

In [11], Tkachuk and R. G. Wilson use the idea of Bella and Spadaro to define the class of cellular-compact spaces: They call a space  $X$  *cellular-compact* if for any disjoint family  $\mathcal{U}$  of non-empty open subsets of  $X$ ,

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2010 *Mathematics Subject Classification.* Primary: 54C35, 54C05; Secondary: 46A50.

*Key words and phrases.* cellular-compact space, countably compact space, dense subspace, function space,  $\omega$ -bounded space,  $P$ -space, pseudocompact space, pseudo-complete space.

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