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TOPOLOGY PROCEEDINGS

Volume 56, 2020

Pages 147–159

http://topology.nipissingu.ca/tp/

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Electronically published on October 21, 2019

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Volume 56 (2020)
Pages 147-159

E-Published on October 21, 2019

CELLULAR COMPACTNESS IN FUNCTION SPACES

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Abstract. Given a Tychonoff space X, we provide some necessary and sufficient conditions (in terms of the topology of the space X) for $C_p(X,[0,1])$ to be cellular-compact. We show that countable compactness of $C_p(X,[0,1])$ implies its cellular compactness, but cellular compactness of $C_p(X,[0,1])$ does not imply existence of a dense countably compact subspace in the space $C_p(X,[0,1])$. We also establish that $C_p(X)$ is σ-cellular-compact if and only if X is finite. Besides, pseudocompleteness of $C_p(X)$ implies that $C_p(X,[0,1])$ is cellular-compact.

1. Introduction

Angelo Bella and Santi Spadaro introduce in [1] the class of cellular-Lindelöf spaces. Recall that a space X is *cellular-Lindelöf* if for any disjoint family $\mathcal{U}$ of non-empty open subsets of X, there exists a Lindelöf subspace $L \subset X$ such that $L \cap U \neq \emptyset$ for any $U \in \mathcal{U}$. Wei-Feng Xuan and Yan-Kui Song construct in [12] an example of a weakly Lindelöf space that is not cellular-Lindelöf. V. V. Tkachuk proves in [10] that cellular-Lindelöf spaces need not be weakly Lindelöf, while Bella and Spadaro establish in [2] that every cellular-Lindelöf monotonically normal space is Lindelöf and prove, under $2^{<\mathfrak{c}} = \mathfrak{c}$, that every normal cellular-Lindelöf first countable space has cardinality not greater than $\mathfrak{c}$.

In [11], Tkachuk and R. G. Wilson use the idea of Bella and Spadaro to define the class of cellular-compact spaces: They call a space X *cellular-compact* if for any disjoint family $\mathcal{U}$ of non-empty open subsets of X,

2010 *Mathematics Subject Classification.* Primary: 54C35, 54C05; Secondary: 46A50.

Key words and phrases. cellular-compact space, countably compact space, dense subspace, function space, ω-bounded space, P-space, pseudocompact space, pseudocomplete space.