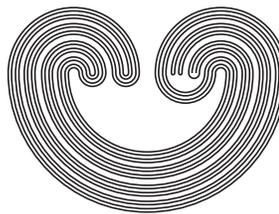


<http://topology.auburn.edu/tp/>

TOPOLOGY PROCEEDINGS



Volume 56, 2020

Pages 147–159

<http://topology.nipissingu.ca/tp/>

CELLULAR COMPACTNESS IN FUNCTION SPACES

by

VLADIMIR V. TKACHUK

Electronically published on October 21, 2019

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings

Department of Mathematics & Statistics

Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

CELLULAR COMPACTNESS IN FUNCTION SPACES

VLADIMIR V. TKACHUK

ABSTRACT. Given a Tychonoff space X , we provide some necessary and sufficient conditions (in terms of the topology of the space X) for $C_p(X, [0, 1])$ to be cellular-compact. We show that countable compactness of $C_p(X, [0, 1])$ implies its cellular compactness, but cellular compactness of $C_p(X, [0, 1])$ does not imply existence of a dense countably compact subspace in the space $C_p(X, [0, 1])$. We also establish that $C_p(X)$ is σ -cellular-compact if and only if X is finite. Besides, pseudocompleteness of $C_p(X)$ implies that $C_p(X, [0, 1])$ is cellular-compact.

1. INTRODUCTION

Angelo Bella and Santi Spadaro introduce in [1] the class of cellular-Lindelöf spaces. Recall that a space X is *cellular-Lindelöf* if for any disjoint family \mathcal{U} of non-empty open subsets of X , there exists a Lindelöf subspace $L \subset X$ such that $L \cap U \neq \emptyset$ for any $U \in \mathcal{U}$. Wei-Feng Xuan and Yan-Kui Song construct in [12] an example of a weakly Lindelöf space that is not cellular-Lindelöf. V. V. Tkachuk proves in [10] that cellular-Lindelöf spaces need not be weakly Lindelöf, while Bella and Spadaro establish in [2] that every cellular-Lindelöf monotonically normal space is Lindelöf and prove, under $2^{<\mathfrak{c}} = \mathfrak{c}$, that every normal cellular-Lindelöf first countable space has cardinality not greater than \mathfrak{c} .

In [11], Tkachuk and R. G. Wilson use the idea of Bella and Spadaro to define the class of cellular-compact spaces: They call a space X *cellular-compact* if for any disjoint family \mathcal{U} of non-empty open subsets of X ,

2010 *Mathematics Subject Classification.* Primary: 54C35, 54C05; Secondary: 46A50.

Key words and phrases. cellular-compact space, countably compact space, dense subspace, function space, ω -bounded space, P -space, pseudocompact space, pseudo-complete space.

©2019 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.