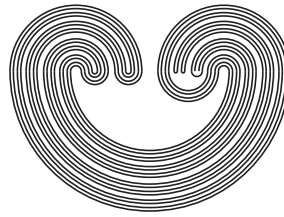


<http://topology.auburn.edu/tp/>

TOPOLOGY PROCEEDINGS



Volume 57, 2021

Pages 1–14

<http://topology.nipissingu.ca/tp/>

TOPOLOGICAL MONOIDS ARE TRANSFINITELY π_1 -COMMUTATIVE AT THE IDENTITY ELEMENT

by

JEREMY BRAZAS AND PATRICK GILLESPIE

Electronically published on April 21, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

TOPOLOGICAL MONOIDS ARE TRANSFINITELY π_1 -COMMUTATIVE AT THE IDENTITY ELEMENT

JEREMY BRAZAS AND PATRICK GILLESPIE

ABSTRACT. Infinite products and infinite products of commutators play an important role in the homotopy theory of Peano continua and other locally path-connected spaces. In this paper, we identify an analogue of the Eckmann-Hilton Principle that applies to infinite products in fundamental groups of topological monoids and slightly more general objects called pre- Δ -monoids. In particular, we show that every pre- Δ -monoid M is “transfinitely π_1 -commutative” in the sense that permutation of the factors of any infinite loop-concatenation indexed by a countably infinite order and based at the identity $e \in M$ is a homotopy invariant action.

1. INTRODUCTION

The Eckmann-Hilton Principle [4] states that if a set M is equipped with two unital binary operations $*$ and \cdot satisfying the distributive law $(a \cdot b) * (c \cdot d) = (a * c) \cdot (b * d)$, then the operations $*$ and \cdot agree and are associative and commutative. Applying this principle to the fundamental group $\pi_1(X, e)$ of any H -space (X, e) , it follows that $\pi_1(X, e)$ is abelian. Since every loop space $\Omega(X, e)$ is an H -space, one has as a corollary that all higher homotopy groups are abelian.

In the homotopy theory of Peano continua and other “wild” spaces, infinite product operations on homotopy classes play an important role.

2010 *Mathematics Subject Classification.* Primary 57M05; Secondary 08A65, 55Q52.

Key words and phrases. Infinitely commutative, infinite product, transfinite product, fundamental group, topological monoid.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.