http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 57, 2021

Pages 25–36

http://topology.nipissingu.ca/tp/

Cusp transitivity in hyperbolic 3-manifolds

by

Roger Vogeler

Electronically published on June 10, 2020

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 57 (2021)
Pages 25-36

E-Published on June 10, 2020

CUSP TRANSITIVITY IN HYPERBOLIC 3-MANIFOLDS

ROGER VOGELER

Abstract. Let M be a cusped finite-volume hyperbolic three-manifold with isometry group G. Then G induces a k-transitive action by permutation on the cusps of M for some integer $k \geq 0$. Generically G is trivial and $k = 0$, but $k > 0$ does occur in special cases. We show examples with $k = 1, 2, 4$. An interesting question concerns the possible number of cusps for a fixed k. Our main result provides an answer for $k = 2$ by constructing a family of manifolds having no upper bound on the number of cusps.

1. Introduction

An action of a group G on a set S is called k-transitive if, for every choice of distinct elements $x_1, ..., x_k \in S$ and every choice of distinct targets $y_1, ..., y_k \in S$, there is an element $g \in G$ such that $g(x_i) = y_i$. The term *transitive* means 1-transitive; actions with $k > 1$ are *multiply transitive*. The number of elements in S is the *degree*. Transitive actions are common (for example, every group acts transitively on itself by left multiplication), while multiply-transitive actions are relatively rare. The theory is well developed; see [4].

It is obvious that the isometry group of a complete finite-volume hyperbolic three-manifold induces a permutation action on the set of cusps. In this paper we call such a manifold *k-transitive* if the induced action is k-transitive. Note that this definition is of the 'inclusive hierarchy' type. For instance, a 3-transitive manifold is automatically 2-transitive, and possibly 4-transitive as well.

Kojima [8] shows by construction that every finite group G occurs as the isometry group of some closed hyperbolic 3-manifold. At one stage,

2020 *Mathematics Subject Classification.* 57M50, 57M60, 57M25.

Key words and phrases. Hyperbolic 3-manifold, cusp, multiple transitivity, pseudo-Anosov.