http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 57, 2021

Pages 61–76

http://topology.nipissingu.ca/tp/

Selectivity properties of spaces

by

William Chen-Mertens and Paul J. Szeptycki

Electronically published on June 10, 2020

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 57 (2021)
Pages 61-76

E-Published on June 10, 2020

SELECTIVITY PROPERTIES OF SPACES

WILLIAM CHEN-MERTENS AND PAUL J. SZEPTYCKI

Abstract. This paper addresses several questions of Feng, Gruenhage, and Shen which arose from Michael's theory of continuous selections from countable spaces. We construct an example of a space which is $(\omega+1)$-selective but not $\mathbb{Q}$-selective from $\mathfrak{d}=\omega_1$, and an $(\omega+1)$-selective space which is not selective for a P-point ultrafilter from the assumption of CH. We also produce ZFC examples of Fréchet spaces where countable subsets are first countable which are not $(\omega+1)$-selective.

1. Introduction

Suppose X, Y are topological spaces and $\varphi : Y \to \mathcal{P}(X) \setminus \{\emptyset\}$ is a map. A general question investigated in detail by Michael asks under what conditions it is possible to find a continuous $s : Y \to X$ so that $s(y) \in \varphi(y)$ for all $y \in Y$. It is natural to require some kind of continuity assumption for φ. So let $\mathcal{F}(X)$ be the set of all nonempty closed subsets of X equipped with the Vietoris topology generated by the sets

(1) $\{A \in \mathcal{F}(X) : A \cap W \neq \emptyset\}$
(2) $\{A \in \mathcal{F}(X) : A \subseteq W\}$

where W ranges through open subsets of X. A map $\varphi : Y \to \mathcal{F}(X)$ is called *lower semicontinuous* if it is continuous with respect to open sets of the first kind, i.e., if for every nonempty open $W \subseteq X$, the set $\{y \in Y : \varphi(y) \cap W \neq \emptyset\}$ is open in Y.

2020 *Mathematics Subject Classification.* Primary 54A20; Secondary 03E57, 03E35.

Key words and phrases. Set-valued mapping, continuous selection, $(\omega+1)$-selective, C-selective, tight gap, Fréchet space.