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TOPOLOGY PROCEEDINGS

Volume 57, 2021

Pages 77–86

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Matthew C. B. Zaremsky

Electronically published on June 13, 2020

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 57 (2021)
Pages 77-86

E-Published on June 13, 2020

A SHORT ACCOUNT OF WHY THOMPSON'S GROUP F IS OF TYPE F_∞

MATTHEW C. B. ZAREMSKY

Abstract. In 1984 Brown and Geoghegan proved that Thompson's group F is of type F_∞, making it the first example of an infinite dimensional torsion-free group of type F_∞. Over the decades a different, shorter proof has emerged, which is more streamlined and generalizable to other groups. It is difficult, however, to isolate this proof in the literature just for F itself, with no complicated generalizations considered and no additional properties proved. The goal of this expository note then is to present the "modern" proof that F is of type F_∞, and nothing else.

Introduction and History. A *classifying space* for a group G is a CW complex Y with $\pi_1(Y) \cong G$ and $\pi_k(Y) = 0$ for all $k \neq 1$. If G admits a classifying space with finite n-skeleton, we say G is of *type F_n*. Equivalently, G is of type F_n if it admits a free, cocompact, cellular action on an $(n-1)$-connected CW complex. Being of type F_1 is equivalent to being finitely generated, and being of type F_2 is equivalent to being finitely presented. We say G is of *type F_∞* if it is of type F_n for all n. Thompson's group F was the first example of a torsion-free group of type F_∞ with no finite dimensional classifying space. Indeed, F cannot have a finite dimensional classifying space since it turns out to contain infinite rank free abelian subgroups, and so in some sense is an "infinite dimensional" group.

The fastest definition of F is via the infinite presentation

$$F = \langle x_0, x_1, x_2, \cdots \mid x_j x_i = x_i x_{j+1} \text{ for all } i < j \rangle.$$

2020 *Mathematics Subject Classification.* Primary 20F65; Secondary 57M07.
Key words and phrases. Thompson's group, finiteness properties, classifying space.