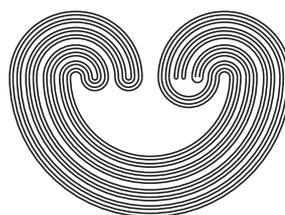


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COARSE SPACES, ULTRAFILTERS AND DYNAMICAL SYSTEMS

by

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COARSE SPACES, ULTRAFILTERS AND DYNAMICAL SYSTEMS

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ABSTRACT. For a coarse space (X, \mathcal{E}) , X^\sharp denotes the set of all unbounded ultrafilters on X endowed with the parallelity relation: $p \parallel q$ if there exists $E \in \mathcal{E}$ such that $E[P] \in q$ for each $P \in p$. If (X, \mathcal{E}) is finitary then there exists a group G of permutations of X such that the coarse structure \mathcal{E} has the base $\{(x, gx) : x \in X, g \in G\} : F \in [G]^{<\omega}, id \in F\}$. We survey and analyze interplays between (X, \mathcal{E}) , X^\sharp and the dynamical system (G, X^\sharp) .

The dynamical Švarc-Milnor Theorem and Gromov Theorem arose at the dawn of *Geometric Group Theory*. In both cases, a group or a pair of groups act on some locally compact spaces, see [22, Chapter 1]. The Gromov coupling criterion was transformed into the powerful tool in coarse equivalences (see references in [23]), however some natural questions on the coarse equivalence of groups need more delicate combinatorial technique, see [4].

In this paper, we describe and survey the dynamical approach to coarse spaces originated in the algebra of the Stone-Ćech compactification. We identify the Stone-Ćech compactification βG of a discrete group G with the set of all ultrafilters on G . The left regular action G on G gives rise to the action of G on βG by $(g, p) \mapsto gp, gp = \{gP : P \in p\}$. In turn, the dynamical system $(G, \beta G)$ induces on βG the structure of a right topological semigroup. The product pq of ultrafilters p, q is defined by $A \in pq$ if and only if $\{g \in G : g^{-1}A \in q\} \in p$. The semigroup βG has a very rich algebraic structure and plenty of combinatorial applications; see nice paper [5], capital book [6] or booklet [9].

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