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THE CONNECTED COUNTABLE SPACES OF BING AND RITTER ARE TOPOLOGICALLY HOMOGENEOUS

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Abstract. Answering a problem posed by the second author on Mathoverflow [1], we prove that the connected countable Hausdorff spaces constructed by Bing [5] and Ritter [19] are topologically homogeneous.

In [5] Bing presented a simple construction of a Hausdorff space which is countable and connected. This example is included to the book "Counterexamples in Topology" [22, Ex.75]. The first (difficult) example of a connected countable Hausdorff space was constructed by Urysohn in [26]. For some other examples of such spaces, see [2], [3], [4], [6], [7], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22, Ex. 61,126], [23], [24], [25].

The Bing space $\mathbb{B}$ is the rational half-plane $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : y \geq 0\}$ endowed with the topology τ consisting of the sets $U \subseteq \mathbb{B}$ such that for every point $(a, b) \in U$ there exists $\varepsilon > 0$ such that

$$\{(x,0) \in \mathbb{B} : |x-(a-b/\sqrt{3})| < \varepsilon\} \cup \{(x,0) \in \mathbb{B} : |x-(a+b/\sqrt{3})| < \varepsilon\} \subseteq U.$$

Observe that the points $(a - b/\sqrt{3}, 0)$ and $(a + b/\sqrt{3}, 0)$ are the base vertices of the equilateral triangle with vertex at (a, b) and base on the line $\mathbb{R} \times \{0\}$.

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