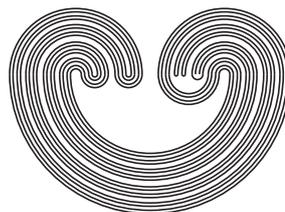


<http://topology.auburn.edu/tp/>

TOPOLOGY PROCEEDINGS



Volume 57, 2021

Pages 149–158

<http://topology.nipissingu.ca/tp/>

THE CONNECTED COUNTABLE SPACES OF BING AND RITTER ARE TOPOLOGICALLY HOMOGENEOUS

by

IRYNA BANAKH, TARAS BANAKH, OLENA HRYNIV,
AND YARYNA STELMAKH

Electronically published on July 21, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

THE CONNECTED COUNTABLE SPACES OF BING AND RITTER ARE TOPOLOGICALLY HOMOGENEOUS

IRYNA BANAKH, TARAS BANAKH, OLENA HRYNIV,
AND YARYNA STELMAKH

ABSTRACT. Answering a problem posed by the second author on Mathoverflow [1], we prove that the connected countable Hausdorff spaces constructed by Bing [5] and Ritter [19] are topologically homogeneous.

In [5] Bing presented a simple construction of a Hausdorff space which is countable and connected. This example is included to the book “Counterexamples in Topology” [22, Ex.75]. The first (difficult) example of a connected countable Hausdorff space was constructed by Urysohn in [26]. For some other examples of such spaces, see [2], [3], [4], [6], [7], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22, Ex. 61,126], [23], [24], [25].

The Bing space \mathbb{B} is the rational half-plane $\{(x, y) \in \mathbb{Q} \times \mathbb{Q} : y \geq 0\}$ endowed with the topology τ consisting of the sets $U \subseteq \mathbb{B}$ such that for every point $(a, b) \in U$ there exists $\varepsilon > 0$ such that

$$\{(x, 0) \in \mathbb{B} : |x - (a - b/\sqrt{3})| < \varepsilon\} \cup \{(x, 0) \in \mathbb{B} : |x - (a + b/\sqrt{3})| < \varepsilon\} \subseteq U.$$

Observe that the points $(a - b/\sqrt{3}, 0)$ and $(a + b/\sqrt{3}, 0)$ are the base vertices of the equilateral triangle with vertex at (a, b) and base on the line $\mathbb{R} \times \{0\}$.

2020 *Mathematics Subject Classification.* 54D05, 54D10.

Key words and phrases. Bing space, topologically homogeneous.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.