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by

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SOME PROPERTIES OF CARTESIAN PRODUCTS AND STONE-ČECH COMPACTIFICATIONS

NEIL HINDMAN AND DONA STRAUSS

Abstract. Given a discrete space S, the Stone-Čech compactification βS of S consists of all of the ultrafilters on S. If $p \in \beta S$ and $q \in \beta T$, then the *tensor product*, $p \otimes q \in \beta(S \times T)$. If $(S, \cdot)$ is a semigroup and $p, q \in \beta S$, then $p \otimes q$ is intimately related to the algebraic product $p \cdot q$. We investigate tensor products in this paper, showing among other things, that tensor products are topologically rare. For example, $S^* \otimes T^*$ is nowhere dense in $\beta(S \times T)$, where $S^* = \beta S \setminus S$.

We also investigate Cartesian products of Stone-Čech compactifications, considering the question of whether, given semigroups $(S, \cdot)$ and $(T, \cdot)$, $(\beta S)^u$ and $(\beta T)^v$ can be isomorphic for distinct positive integers u and v. We obtain conditions guaranteeing that the answer is "no" as well as some examples where the answer is "yes".

1. Introduction

The tensor product of two ultrafilters is a special case of the notion of the *sum of ultrafilters* introduced by Frolík in paragraph 1.2 of [7].

Definition 1.1. Let S and T be discrete spaces, let $p \in \beta S$, and let $q \in \beta T$. Then the *tensor product of p and q* is defined by

$$p \otimes q = \{A \subseteq S \times T : \{x \in S : \{y \in T : (x, y) \in A\} \in q\} \in p\}.$$

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