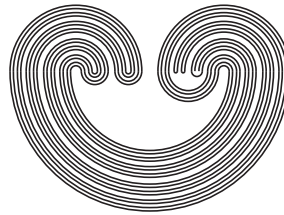


<http://topology.auburn.edu/tp/>

TOPOLOGY PROCEEDINGS



Volume 57, 2021

Pages 279–304

<http://topology.nipissingu.ca/tp/>

SOME PROPERTIES OF CARTESIAN PRODUCTS AND STONE-ČECH COMPACTIFICATIONS

by

NEIL HINDMAN AND DONA STRAUSS

Electronically published on November 1, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

SOME PROPERTIES OF CARTESIAN PRODUCTS AND STONE-ĆECH COMPACTIFICATIONS

NEIL HINDMAN AND DONA STRAUSS

ABSTRACT. Given a discrete space S , the Stone-Ćech compactification βS of S consists of all of the ultrafilters on S . If $p \in \beta S$ and $q \in \beta T$, then the *tensor product*, $p \otimes q \in \beta(S \times T)$. If (S, \cdot) is a semigroup and $p, q \in \beta S$, then $p \otimes q$ is intimately related to the algebraic product $p \cdot q$. We investigate tensor products in this paper, showing among other things, that tensor products are topologically rare. For example, $S^* \otimes T^*$ is nowhere dense in $\beta(S \times T)$, where $S^* = \beta S \setminus S$.

We also investigate Cartesian products of Stone-Ćech compactifications, considering the question of whether, given semigroups (S, \cdot) and (T, \cdot) , $(\beta S)^u$ and $(\beta T)^v$ can be isomorphic for distinct positive integers u and v . We obtain conditions guaranteeing that the answer is “no” as well as some examples where the answer is “yes”.

1. INTRODUCTION

The tensor product of two ultrafilters is a special case of the notion of the *sum of ultrafilters* introduced by Frolík in paragraph 1.2 of [7].

Definition 1.1. Let S and T be discrete spaces, let $p \in \beta S$, and let $q \in \beta T$. Then the *tensor product of p and q* is defined by

$$p \otimes q = \{A \subseteq S \times T : \{x \in S : \{y \in T : (x, y) \in A\} \in q\} \in p\}.$$

2020 *Mathematics Subject Classification.* Primary 54D35; Secondary 54D80, 22A15.

Key words and phrases. Cartesian product, tensor product, semigroup, smallest ideal.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.