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Rectangles Inscribed in Locally Connected Plane Continua

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RECTANGLES INSCRIBED IN LOCALLY CONNECTED PLANE CONTINUA

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Abstract. A plane continuum X is said to admit an inscribed rectangle if for every embedding $\gamma : X \to \mathbb{R}^2$, all vertices of at least one Euclidean rectangle lie on $\gamma(X)$. In this paper, we prove that if a plane continuum X contains a copy of the capital letter H continuum, the simple 4-od, or S^1, then X admits an inscribed rectangle. Also, we prove that the only locally connected plane continua that do not admit an inscribed rectangle are the arc and the simple 3-od.

1. Introduction

In 1911, Otto Toeplitz [12] asked, Does every Jordan curve admit an inscribed square? This question is known as the square peg problem and it is still an open question. Since then, a lot of work has been done in trying to answer it. Benjamin Matschke [8] presents a general overview of the work that has been done so far. In 2017, Terence Tao [11] presented, using integration, an approach to solve the famous problem.

There are many problems related to the square peg problem (see [4], [6], [8], [10]). In particular, one may ask whether S^1 admits an inscribed rectangle. If one does not prescribe the aspect ratio of the rectangle, the answer to this question is affirmative. This answer is due to H. Vaughan (see [7, p. 71]); he used the second symmetric product of S^1. In [2]

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