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ON BELLAMY'S SET FUNCTION Γ

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Abstract. David P. Bellamy defined a set function Γ in his paper *Some topics in modern continua theory* [Continua, Decompositions, Manifolds, Austin, TX: University of Texas Press, 1983, 1–26]. We investigate this set function and prove his conjecture.

1. Introduction

David P. Bellamy defined a set function Γ in [1, p. 11]. He mentions that he has not done a serious study of this set function and he conjectures that [1, Theorem 1] is true for Γ. We show in Theorem 3.30 that this is the case.

The paper is divided in five sections. We provide definitions in section 2. In section 3, we present properties of Γ. For example, in Theorem 3.3, we characterize compacta for which $\Gamma(\emptyset) = \emptyset$. In particular, for a continuum, we have that $\Gamma(\emptyset) = \emptyset$ (see Corollary 3.4). We also characterize continua X such that $\Gamma(A) = A$ for all closed subsets A of X (see Theorem 3.20). In section 4, we consider the symmetry, additivity, and idempotency of Γ. For a continuum X, we characterize the additivity of Γ in terms of families of closed subsets of X whose union is closed in X (see Theorem 4.5). We also prove, in Theorem 4.17, that Γ is not idempotent on the product of an indecomposable continuum and a continuum. In section 5, we study properties of Γ when its restriction to the hyperspace of nonempty closed subsets is continuous. We show in Theorem 5.1 that

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