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ON RATIOS OF HOMOTOPY AND HOMOLOGY RANKS OF FIBRATIONS

TOSHIHIRO YAMAGUCHI AND SHOJI YOKURA

Abstract. For a simply connected CW complex X, we let $h(X) = \dfrac{\dim(\pi_*(X)\otimes\mathbb{Q})}{\dim H_*(X;\mathbb{Q})}$. In this paper, we propose to evaluate $h(X)$ of the total space X of a fibration $\xi : F \hookrightarrow X \to B$ of elliptic spaces by $h(F)$, $h(B)$, and $h(F \times B)$. A conjectural formula is

$$\frac{1}{2}\cdot h(F\times B) \leqq h(X) < h(F) + h(B) + \frac{1}{4}.$$

1. Introduction

The homotopy and homology ranks of a topological space X are defined by $\dim(\pi_*(X)\otimes\mathbb{Q})$ and $\dim H_*(X;\mathbb{Q})$, respectively, where

$$\pi_*(X)\otimes\mathbb{Q} := \sum_{k\geqq 1}\pi_k(X)\otimes\mathbb{Q} \quad \text{and} \quad H_*(X;\mathbb{Q}) := \sum_{k\geqq 0} H_k(X;\mathbb{Q}).$$

Definition 1.1. Let X be a simply connected CW complex with $\dim H_*(X;\mathbb{Q}) < \infty$. Then we define

$$h(X) := \frac{\dim(\pi_*(X)\otimes\mathbb{Q})}{\dim H_*(X;\mathbb{Q})}.$$

If X is a hyperbolic space, i.e., $\dim(\pi_*(X)\otimes\mathbb{Q}) = \infty$, then we set $h(X) := \infty$.

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