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FINITE GRAPHS AND INVERSE LIMITS WITH SET-VALUED FUNCTIONS ON INTERVALS

VAN NALL AND IVON VIDAL-ESCOBAR

Abstract. If the inverse limit of upper semi-continuous set-valued functions from $[0,1]$ to the closed subsets of $[0,1]$ is a finite graph G, then for some N and all $n \geq N$, the projection onto the first n coordinates of that inverse limit is a projection onto a finite graph that is homeomorphic to G.

1. Introduction

It has not been shown that every finite graph can be obtained as an inverse limit with set-valued functions on $[0,1]$, though it is known that the only finite graph that is such a generalized inverse limit with a single set-valued function is an arc [7]. In this paper, we are considering inverse limits with different set-valued functions from $[0,1]$ to the closed subsets of $[0,1]$. For example, if n is a natural number such that for each natural number $i \leq n$, we define $f_i(x) = \{0, x\}$ for $0 \leq x < 1$ and $f(1) = [0,1]$, and then for $i > n$, we define $f_i(x) = \{x\}$ for all $0 \leq x \leq 1$, then the inverse limit with this sequence of set-valued functions is a complete graph with $n+2$ vertices. Any planar graph can be expressed as an inverse limit with a sequence $\{f_i\}$ of set-valued functions where the graph of f_1 is an imbedding of the planar graph in $[0,1] \times [0,1]$, and for $i > 1$, $f_i(x) = \{x\}$ for each $x \in [0,1]$. The complete m and n bipartite graphs can be expressed as an inverse limit with different set-valued functions f_1 and f_2 where the graph of f_1 is the union of straight lines connecting the n points $\{(\frac{i}{n-1}, 0) : i = 0 \dots n-1\}$ to the single point $(\frac{1}{2}, 1)$, f_2 is the union of straight lines connecting the m points $\{(0, \frac{i}{m-1}) : i = 0 \dots m-1\}$ to

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