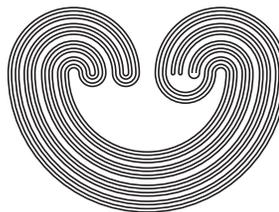


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## FINITE GRAPHS AND INVERSE LIMITS WITH SET-VALUED FUNCTIONS ON INTERVALS

by

VAN NALL AND IVON VIDAL-ESCOBAR

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## FINITE GRAPHS AND INVERSE LIMITS WITH SET-VALUED FUNCTIONS ON INTERVALS

VAN NALL AND IVON VIDAL-ESCOBAR

**ABSTRACT.** If the inverse limit of upper semi-continuous set-valued functions from  $[0, 1]$  to the closed subsets of  $[0, 1]$  is a finite graph  $G$ , then for some  $N$  and all  $n \geq N$ , the projection onto the first  $n$  coordinates of that inverse limit is a projection onto a finite graph that is homeomorphic to  $G$ .

### 1. INTRODUCTION

It has not been shown that every finite graph can be obtained as an inverse limit with set-valued functions on  $[0, 1]$ , though it is known that the only finite graph that is such a generalized inverse limit with a single set-valued function is an arc [7]. In this paper, we are considering inverse limits with different set-valued functions from  $[0, 1]$  to the closed subsets of  $[0, 1]$ . For example, if  $n$  is a natural number such that for each natural number  $i \leq n$ , we define  $f_i(x) = \{0, x\}$  for  $0 \leq x < 1$  and  $f_i(1) = [0, 1]$ , and then for  $i > n$ , we define  $f_i(x) = \{x\}$  for all  $0 \leq x \leq 1$ , then the inverse limit with this sequence of set-valued functions is a complete graph with  $n + 2$  vertices. Any planar graph can be expressed as an inverse limit with a sequence  $\{f_i\}$  of set-valued functions where the graph of  $f_1$  is an imbedding of the planar graph in  $[0, 1] \times [0, 1]$ , and for  $i > 1$ ,  $f_i(x) = \{x\}$  for each  $x \in [0, 1]$ . The complete  $m$  and  $n$  bipartite graphs can be expressed as an inverse limit with different set-valued functions  $f_1$  and  $f_2$  where the graph of  $f_1$  is the union of straight lines connecting the  $n$  points  $\{(\frac{i}{n-1}, 0) : i = 0 \dots n-1\}$  to the single point  $(\frac{1}{2}, 1)$ ,  $f_2$  is the union of straight lines connecting the  $m$  points  $\{(0, \frac{i}{m-1}) : i = 0 \dots m-1\}$  to

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