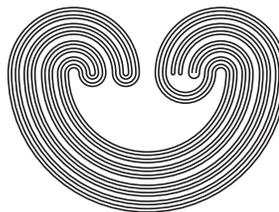


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## COMPLETENESS OF LEXICOGRAPHIC PRODUCTS OF GO-SPACES

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## COMPLETENESS OF LEXICOGRAPHIC PRODUCTS OF GO-SPACES

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**ABSTRACT.** Variations of Dedekind completeness of lexicographic products of GO-spaces are studied. As a corollary, we see that whenever  $\gamma$  is limit and all GO-spaces  $X_\alpha$ 's have minimal elements but have no maximal elements, connectedness of a lexicographic product  $\prod_{\alpha < \gamma} X_\alpha$  implies *non*-connectedness of all  $X_\alpha$ 's.

### 1. INTRODUCTION

All spaces are assumed to be regular  $T_1$  and when we consider a product  $\prod_{\alpha < \gamma} X_\alpha$ , all  $X_\alpha$  are assumed to have cardinality at least 2 with  $\gamma \geq 2$ . Set theoretical and topological terminology follow [9] and [1]. The following are well known:

- a LOTS  $X$  is compact if and only if every subset  $A$  of  $X$  has a least upper bound  $\sup_X A$ , where  $\sup_X \emptyset$  is defined to be the minimal element  $\min X$  of  $X$ ; see [1, Problem 3.12.3(a)];
- a lexicographic product  $\prod_{\alpha < \gamma} X_\alpha$  of LOTs is compact if and only if all  $X_\alpha$ 's are compact; see [2, Theorem 4.2.1].

Obviously, a LOTS  $X$  is compact if and only if both of the following properties hold:

- (a) every *non-empty* subset  $A$  of  $X$  has a least upper bound  $\sup_X A$ ;
- (b) every *non-empty* subset  $A$  of  $X$  has a greatest lower bound  $\inf_X A$ .

One might conjecture that if a LOTS  $X_\alpha$  satisfies property (a) for every  $\alpha < \gamma$ , then the lexicographic product  $\prod_{\alpha < \gamma} X_\alpha$  also satisfies property

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