http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 58, 2021

Pages 183–206

http://topology.nipissingu.ca/tp/

Central Sets Theorem for Arbitrary Adequate Partial Semigroups

by

Neil Hindman and Kendra Pleasant

Electronically published on November 13, 2020

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 58 (2021)
Pages 183-206

E-Published on November 13, 2020

CENTRAL SETS THEOREM FOR ARBITRARY ADEQUATE PARTIAL SEMIGROUPS

NEIL HINDMAN AND KENDRA PLEASANT

Abstract. We establish a Central Sets Theorem valid for arbitrary adequate partial semigroups. Except for the requirement that the sequences considered be *adequate*, it is identical to the currently most general version of the Central Sets Theorem for semigroups. We present an application to the partial semigroup of located words and obtain several related results.

1. Introduction

Since their introduction in 1981 in [3], central sets in semigroups have been shown to have significant combinatorial properties. And since, whenever a semigroup is partitioned into finitely many sets, one of those sets must be central, there are important consequences for Ramsey theory. For example, it is shown in [3] that if C is a central subset of $\mathbb{N}$, then C contains solutions to every partition regular system of homogeneous linear equations. See the survey [6] for many more examples of properties enjoyed by any central set. And see section 2 of this paper for definitions of this or any other unfamiliar notions discussed in this introduction.

Most of the desirable properties enjoyed by central sets are consequences of the Central Sets Theorem. The original version established in [3] is the following. (Given a set X, we write $\mathcal{P}_f(X)$ for the set of finite nonempty subsets of X.)

Theorem 1.1. *Let F be a finite set of sequences in $\mathbb{Z}$ and let C be central in $\mathbb{N}$. There exist a sequence $\langle a_n\rangle_{n=1}^{\infty}$ in $\mathbb{N}$ and a sequence $\langle H_n\rangle_{n=1}^{\infty}$ in $\mathcal{P}_f(\mathbb{N})$ such that*

2020 *Mathematics Subject Classification.* Primary 54D35, 22A30; Secondary 05D10.

Key words and phrases. Central Sets Theorem, partial semigroups.