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A GAME DIMENSION FUNCTION

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ABSTRACT. We define a topological *game dimension*, (gd), such that $gd(X) = Ind(X)$ where X is a hereditarily normal space and $Ind(X) \leq gd(X)$ where X is a normal space. We ask a question which is about whether $gd(X) = Ind(X)$ where X is a normal space. By making some appropriate modifications in this game, the dimension functions Ind and ind can be characterized in the realm of normal spaces.

1. Introduction

Dimension theory (see [4]) and topological games (see [6] or [1]) are interesting and important topics in general topology. Some relations between topological games and dimension of spaces are discussed in [2] and [3] in the realm of separable metrizable spaces and topological groups. In this paper, by defining a topological game, we characterize the dimension function Ind for hereditarily normal spaces. One can improve this game and characterize the dimension function Ind and ind for normal spaces.

The dimension functions Ind, ind, and dim are basic tools for dimension theory. It is known that for any normal space X, $dimX \leq n \geq 0$ if and only if for every finite sequence $((A_0, B_0), (A_1, B_1), \ldots (A_n, B_n))$ of $n + 1$ pairs disjoint closed subsets of X there exist $L_0, L_1, \ldots, L_n$ such that $L_0 \cap L_1 \cap \ldots \cap L_n = \emptyset$ and L_i is a partition between A_i and B_i (Theorem 3.2.6 [4]). We consider the case where the pairs are given and the partitions are chosen in turn. Let us clear that. Suppose $dimX = 1$; then we know that for any $((A_0, B_0), (A_1, B_1))$ pairs of closed disjoint subsets, there exist partitions L_0 and L_1 such that $L_0 \cap L_1 = \emptyset$. Now, let us be given (A_0, B_0) and we have to choose an L_0 (without seeing (A_1, B_1)).

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