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TOPOLOGY PROCEEDINGS

Volume 58, 2021

Pages 289–301

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Electronically published on May 3, 2021

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 58 (2021)
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E-Published on May 3, 2021

THE KAUFFMAN BRACKET EXPANSION OF A GENERALIZED CROSSING

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Abstract. We examine the Kauffman bracket expansion of the generalized crossing Δ_n, a half-twist on n parallel strands, as an element of the Temperley–Lieb algebra with coefficients in $\mathbb{Z}[A, A^{-1}]$. In particular, we determine the minimum and maximum degrees of all possible coefficients appearing in this expansion. Our main theorem shows that the maximum such degree is quadratic in n, while the minimum such degree is linear. We also include an appendix with explicit expansions for n at most six.

1. Introduction

The Jones polynomial, introduced in [4], had a revolutionary impact on classical knot theory, fundamentally altering the fabric of low-dimensional topology. One well-known method of computing the Jones polynomial is via the Kauffman bracket $\langle\cdot\rangle$, which gives a set of rules for iteratively converting a knot diagram D into an element $\langle D\rangle$ of $\mathbb{Z}[A, A^{-1}]$ [5]. Normalizing this polynomial $\langle D\rangle$ using the writhe of D yields the Jones polynomial of K. More generally, the Kauffman bracket can also be applied to any n-stranded tangle diagram $\mathcal{T}$. In this case, $\langle\mathcal{T}\rangle$ is an element of the Temperley–Lieb algebra TL_n (see [7]) with coefficients in $\mathbb{Z}[A, A^{-1}]$.

The main purpose of this paper is to elicit essential characteristics of the Kauffman bracket of a *generalized crossing* Δ_n, the tangle diagram obtained by performing a half-twist on n parallel unknotted strands. As an element of the braid group, Δ_n is sometimes called the *Garside element*. In [6], Jeffrey Meier and the second author produce a family of

2020 *Mathematics Subject Classification.* Primary 57K14; Secondary 57K10.
Key words and phrases. generalized half-twist, Jones polynomial, Kauffman bracket.
The second author was supported by NSF awards DMS-1664578 and DMS-2005518.