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STEVEN CLONTZ

Abstract. Arhangelskii's properties α_2 and α_4 defined for convergent sequences may be characterized in terms of Scheeper's selection principles. We generalize these results to hold for more general collections and consider these results in terms of selection games.

The following characterizations were given as Definition 1 by Kocinac in [7].

Definition 1. *Arhangelskii's α-principles* $\alpha_i(\mathcal{A}, \mathcal{B})$ are defined as follows for $i \in \{1, 2, 3, 4\}$. Let $A_n \in \mathcal{A}$ for all $n < \omega$; then there exists $B \in \mathcal{B}$ such that:

- α_1: $A_n \cap B$ is cofinite in A_n for all $n < \omega$.
- α_2: $A_n \cap B$ is infinite for all $n < \omega$.
- α_3: $A_n \cap B$ is infinite for infinitely-many $n < \omega$.
- α_4: $A_n \cap B$ is non-empty for infinitely-many $n < \omega$.

When $(\mathcal{A}, \mathcal{B})$ is omitted, it is assumed that $\mathcal{A} = \mathcal{B}$ is the collection $\Gamma_{X,x}$ of sequences converging to some point $x \in X$, as introduced by Arhangelskii in [1]. Provided $\mathcal{A}$ only contains infinite sets, it's easy to see that $\alpha_n(\mathcal{A}, \mathcal{B})$ implies $\alpha_{n+1}(\mathcal{A}, \mathcal{B})$.

We aim to relate these to the following games.

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