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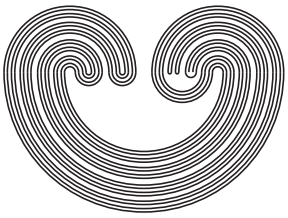

Dominating and pinning down pairs for topological spaces

by

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DOMINATING AND PINNING DOWN PAIRS FOR TOPOLOGICAL SPACES

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Abstract. We call a pair of infinite cardinals (κ, λ) with $\kappa > \lambda$ a *dominating* (resp. *pinning down*) pair for a topological space X if for every subset A of X (resp. family $\mathcal{U}$ of non-empty open sets in X) of cardinality $\leq \kappa$ there is $B \subset X$ of cardinality $\leq \lambda$ such that $A \subset \overline{B}$ (resp. $B \cap U \neq \emptyset$ for each $U \in \mathcal{U}$). Clearly, a dominating pair is also a pinning down pair for X. Our definitions generalize the concepts introduced in [4] resp. [3] which focused on pairs of the form $(2^\lambda, \lambda)$.

The main aim of this paper is to answer a large number of the numerous problems from [4] and [3] that asked if certain conditions on a space X together with the assumption that $(2^\lambda, \lambda)$ or $((2^\lambda)^+, \lambda)$ is a pinning down pair or dominating pair for X would imply $d(X) \leq \lambda$.

This paper is dedicated to the memory of Phil Zenor

1. Introduction

Definition 1.1. Let X be a topological space and let $\kappa > \lambda \geq \omega$ be cardinals.

(i) (κ, λ) is a pinning down pair for X if for every family $\mathcal{U}$ of non-empty open sets in X with $|\mathcal{U}| \leq \kappa$ there is $B \subset X$ of cardinality $\leq \lambda$ such that $B \cap U \neq \emptyset$ for each $U \in \mathcal{U}$. In other words, $\mathcal{U}$ may be pinned down with $\leq \lambda$ points.

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Key words and phrases. density of a topological space, cardinal function, dominating pair, pinning down pair.

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