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by

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BAIRE SPACES, G_δ'S, AND DIRECTED COMPLETE SPACES

WILLIAM FLEISSNER AND LYNNE YENGULALP

Abstract. We discuss the proof of the Baire Category Theorem. We review Čech-completeness, pseudocompleteness, and various forms of α-favorability. We present recent results about subcompactness, directed completeness, the Banach-Mazur game, and the strong Choquet game. A recurring theme is whether a completeness property is inherited by G_δ subspaces. We prove that the class of regular directed complete spaces is exactly the class of G_δ subspaces of subcompact spaces.

1. Introduction and Motivation

We are grateful to have been invited to contribute a paper to the issue of Topology Proceedings honoring Phil Zenor. We appreciate the work that Phil did – not only his mathematical research, but also advising students, organizing conferences, and co-founding this journal.

This paper initially was a 20 minute talk at the AMS Session in San Diego in January 2018. It grew to become a potential colloquium talk, and then grew again to include a survey reviewing recent advances and placing them in context. Some personal view digressions (appearing with the pronoun 'I') appropriate for a colloquium talk remain. There are surveys of completeness properties, in particular [1] and [6], but these need to be updated. Even since [18], there are several new and important results about subcompact spaces, directed complete spaces, and strategies for the Choquet game.

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Key words and phrases. Baire space, α-favorable, Banach-Mazur game, Choquet game, G_δ, subcompact, directed complete.

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