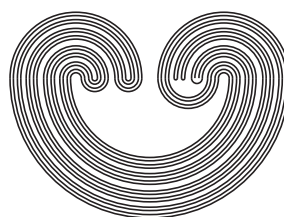


<http://topology.nipissingu.ca/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 59, 2022

Pages 153–162

---

## ON METRIZABLE SUBSETS OF HEREDITARILY NORMAL COMPACT SPACES

by

HEIKKI JUNNILA AND KAZUO TOMOYASU

Electronically published on April 25, 2021

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.nipissingu.ca/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## ON METRIZABLE SUBSETS OF HEREDITARILY NORMAL COMPACT SPACES

HEIKKI JUNNILA AND KAZUO TOMOYASU

**ABSTRACT.** Let  $X$  be a metrizable space which has a hereditarily normal  $\omega_1$ -compactification. We show that  $X$  is rim-separable and that if  $X$  is also connected, then  $w(X) \leq \omega_1$  and  $X$  has a  $\sigma$ -point-finite base by sets with separable boundaries.

### 1. INTRODUCTION

One of the finest contributions of Philip Zenor to the theory of point-set topology was the introduction of the concept of *monotone normality* in 1970 ([13]). This concept turned out to be extremely fruitful and some of the most spectacular results in topology in the last decades deal with monotonically normal spaces.

Also this paper has its roots in work on monotone normality. It is known that many metrizable spaces have monotonically normal compactifications. For example, every locally separable metrizable space has a hereditarily paracompact monotonically normal compactification (see the proof of Proposition 3 below) and this kind of a compactification exists also for every strongly zero-dimensional metrizable space (see [8]). A problem due to P. Gartside asked whether every metrizable space has a monotonically normal compactification ([3, Problem 10]). The problem was solved in [9] by the result that the hedgehog-space  $J(\omega_1)$  admits no monotonically normal compactification.

---

2020 *Mathematics Subject Classification.* 54D35, 54D15, 54D05.

*Key words and phrases.* hereditarily normal,  $\omega_1$ -compact, rim-Lindelöf.

©2021 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.