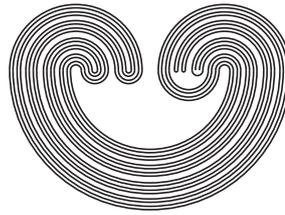


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## MONOTONE ORTHOCOMPACTNESS AND PROPERTY $(A_o)$

by

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PORTER

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## MONOTONE ORTHOCOMPACTNESS AND PROPERTY ( $A_o$ )

GARY GRUENHAGE, STRASHIMIR G. POPVASSILEV, AND JOHN E. PORTER

*Dedicated to the memory of Phillip L. Zenor, our dear colleague, teacher, and friend, with appreciation for his kindness, good cheer, and willingness to always discuss mathematical ideas.*

**ABSTRACT.** We continue the study of monotonic orthocompactness with respect to interior preserving open refinements (abbreviated  $MO_o$ ) introduced by Popvassilev and Porter. We show that a GO-space is  $MO_o$  provided that it contains a  $\sigma$ -closed-discrete set  $D$  such that the complement of its closure is  $MO_o$ ; in particular, Alexandrov's double arrow space as well as the lexicographically ordered square are  $MO_o$ . Hence the result of Chase and Gruenhagen that compact Hausdorff spaces which are monotonically (countably) metacompact must be metrizable does not extend to  $MO_o$ . We show that a compact LOTS which is  $MO_o$  must be first countable, and a monotonically normal space which is  $MO_o$  must be hereditarily paracompact. We also introduce a formally weaker property ( $A_o$ ) as a useful tool in this study. We show that the one-point compactification of an uncountable discrete space, the Alexandrov duplicate of the unit interval, and stationary subsets of regular uncountable cardinals do not have property ( $A_o$ ) and hence are not  $MO_o$ .

### 1. INTRODUCTION

All spaces considered are assumed to be at least  $T_2$ . A space  $X$  is *orthocompact* if every open cover  $\mathcal{U}$  has an interior-preserving open refinement  $\mathcal{V}$  (i.e.,  $\cap \mathcal{W}$  is open for each  $\mathcal{W} \subset \mathcal{V}$ ). It is equivalent to say that there is

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*Key words and phrases.* Monotone covering properties, compact, paracompact, orthocompact, monotone orthocompact, property ( $A_o$ ), GO-spaces, LOTS,  $\sigma$ -closed-discrete dense subset, metrization, stationary sets.

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