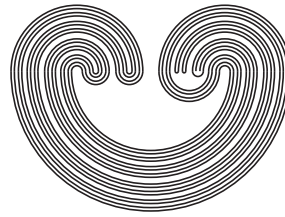


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by

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SOME PROPERTIES OF ONE-POINT EXTENSIONS

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To the memory of Phillip Zenor, a founder of Topology Proceedings

ABSTRACT. A Tychonoff space $X_p = X \cup \{p\}$ is called a one-point extension of X if X is dense in X_p and the remainder $X_p \setminus X$ consists of the singleton $\{p\}$.

We study the following problem: Characterize the spaces X such that **every (some)** one-point extension X_p of X has a given local topological property \mathcal{P} at the point p . The list of properties \mathcal{P} considered in the paper includes, among others: 1) $\{p\}$ is a G_δ -set in X_p ; 2) X_p admits a local countable base at p ; 3) X_p has the Fréchet-Urysohn property at p ; 4) X_p has countable tightness at p .

One of our main results states that a Tychonoff space X is Lindelöf (not pseudocompact) iff the point p is of type G_δ in X_p , for every (for some, respectively) one-point extension X_p of X . We pose several open problems for various concrete properties \mathcal{P} .

1. INTRODUCTION

We consider only Tychonoff spaces. A Tychonoff space X_p is called a *one-point extension* of X if there is a homeomorphic embedding $\pi: X \rightarrow X_p$ such that $\pi(X)$ is dense in X_p and $X_p \setminus \pi(X)$ consists of precisely *one* point, say, p . For simplicity we identify $\pi(X)$ with X , so X is dense in $X_p = X \cup \{p\}$. We will always assume that X is a non-compact space since compact spaces do not admit one-point extensions. Naturally, X admits a compact one-point extension X_p iff X is locally compact.

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Key words and phrases. One-point extension, Stone-Čech compactification, Lindelöf space, character, Fréchet-Urysohn property, G_δ -set, zero-set.

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