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SOME PROPERTIES OF ONE-POINT EXTENSIONS

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To the memory of Phillip Zenor, a founder of Topology Proceedings

Abstract. A Tychonoff space $X_p = X \cup \{p\}$ is called a one-point extension of X if X is dense in X_p and the reminder $X_p \setminus X$ consists of the singleton $\{p\}$.

We study the following problem: Characterize the spaces X such that **every** (**some**) one-point extension X_p of X has a given local topological property $\mathcal{P}$ at the point p. The list of properties $\mathcal{P}$ considered in the paper includes, among others: 1) $\{p\}$ is a G_δ-set in X_p; 2) X_p admits a local countable base at p; 3) X_p has the Fréchet-Urysohn property at p; 4) X_p has countable tightness at p.

One of our main results states that a Tychonoff space X is Lindelöf (not pseudocompact) iff the point p is of type G_δ in X_p, for every (for some, respectively) one-point extension X_p of X. We pose several open problems for various concrete properties $\mathcal{P}$.

1. Introduction

We consider only Tychonoff spaces. A Tychonoff space X_p is called a *one-point extension* of X if there is a homeomorphic embedding $\pi\colon X \to X_p$ such that $\pi(X)$ is dense in X_p and $X_p \setminus \pi(X)$ consists of precisely *one* point, say, p. For simplicity we identify $\pi(X)$ with X, so X is dense in $X_p = X \cup \{p\}$. We will always assume that X is a non-compact space since compact spaces do not admit one-point extensions. Naturally, X admits a compact one-point extension X_p iff X is locally compact.

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