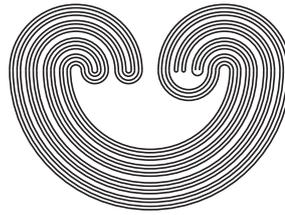


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## ALL PAROVICHENKO SPACES MAY BE SOFT-PAROVICHENKO

by

ALAN DOW AND KLAAS PIETER HART

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## ALL PAROVICHENKO SPACES MAY BE SOFT-PAROVICHENKO

ALAN DOW AND KLAAS PIETER HART

*To the memory of Phil Zenor, one of the founders of this journal*

**ABSTRACT.** It is shown that, assuming the Continuum Hypothesis, every compact Hausdorff space of weight at most  $\mathfrak{c}$  is a remainder in a soft compactification of  $\mathbb{N}$ .

We also exhibit an example of a compact space of weight  $\aleph_1$  — hence a remainder in some compactification of  $\mathbb{N}$  — for which it is consistent that is not the remainder in a soft compactification of  $\mathbb{N}$ .

### INTRODUCTION

A compactification,  $\gamma\mathbb{N}$ , of the discrete space  $\mathbb{N}$  of natural numbers is said to be *soft* if for all pairs  $\langle A, B \rangle$  of disjoint subsets of  $\mathbb{N}$  the following holds: if  $\text{cl } A \cap \text{cl } B \neq \emptyset$  then there is an autohomeomorphism  $h$  of  $\gamma\mathbb{N}$  such that  $h[A] \cap B$  is infinite and  $h$  is the identity on the remainder  $\gamma\mathbb{N} \setminus \mathbb{N}$ .

Banach asked in [1] whether every Parovichenko space is soft-Parovichenko, where a Parovichenko space is defined to be a remainder in some compactification of  $\mathbb{N}$  and, naturally, a soft-Parovichenko space is a remainder in some soft compactification of  $\mathbb{N}$ . Parovichenko's classic theorem, from [7], characterizes, assuming CH, the Parovichenko spaces as the compact Hausdorff spaces of weight at most  $\mathfrak{c}$ .

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