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ALL PAROVICHENKO SPACES MAY BE SOFT-PAROVICHENKO

ALAN DOW AND KLAAS PIETER HART

To the memory of Phil Zenor, one of the founders of this journal

Abstract. It is shown that, assuming the Continuum Hypothesis, every compact Hausdorff space of weight at most $\mathfrak{c}$ is a remainder in a soft compactification of $\mathbb{N}$.

We also exhibit an example of a compact space of weight $\aleph_1$ — hence a remainder in some compactification of $\mathbb{N}$ — for which it is consistent that is not the remainder in a soft compactification of $\mathbb{N}$.

Introduction

A compactification, $\gamma\mathbb{N}$, of the discrete space $\mathbb{N}$ of natural numbers is said to be *soft* if for all pairs $\langle A, B\rangle$ of disjoint subsets of $\mathbb{N}$ the following holds: if $\operatorname{cl} A \cap \operatorname{cl} B \neq \emptyset$ then there is an autohomeomorphism h of $\gamma\mathbb{N}$ such that $h[A] \cap B$ is infinite and h is the identity on the remainder $\gamma\mathbb{N} \setminus \mathbb{N}$.

Banakh asked in [1] whether every Parovichenko space is soft-Parovichenko, where a Parovichenko space is defined to be a remainder in some compactification of $\mathbb{N}$ and, naturally, a soft-Parovichenko space is a remainder in some soft compactification of $\mathbb{N}$. Parovichenko's classic theorem, from [7], characterizes, assuming CH, the Parovichenko spaces as the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

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