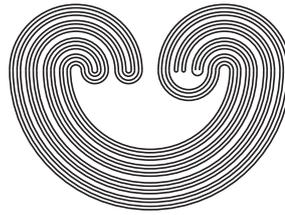


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INEQUALITY AND EQUALITY FOR THE EXTENT OF PRODUCTS WITH A SPECIAL FACTOR

by

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INEQUALITY AND EQUALITY FOR THE EXTENT OF PRODUCTS WITH A SPECIAL FACTOR

YASUSHI HIRATA AND YUKINOBU YAJIMA

ABSTRACT. For a space X , let $e(X) = \sup\{|D| : D \text{ is a closed discrete subset in } X\} \cdot \omega$, which is called the *extent* of X . First, we give some examples of a rectangular product $X \times Y$ with $e(X \times Y) > e(X) \cdot e(Y) = \omega$. Secondly, we give an equivalent condition for a given space X such that $e(X \times Y) > e(X) \cdot e(Y)$ for a certain special factor Y . Finally, we discuss when $e(X \times Y) = e(X) \cdot e(Y)$ for a product $X \times Y$ with a special factor X .

1. INTRODUCTION

For a space X , recall the cardinal function defined by

$$e(X) = \sup\{|D| : D \text{ is a closed discrete subset in } X\} \cdot \omega,$$

which is called the *extent* of X . Obviously, if X is Lindelöf or countably compact, then $e(X) = \omega$.

As is well known, the Sorgenfrey line S is a hereditarily Lindelöf space with $e(S \times S) = 2^\omega > \omega = e(S) \cdot e(S)$. So Shelah raised the following problem in 1978:

Problem 1. Are there two Lindelöf spaces X and Y such that $e(X \times Y) > 2^\omega$?

For this problem, he first proved the following.

Theorem 1.1 (Shelah [24]). *It is consistent that there are Lindelöf spaces X and Y with all points G_δ such that $e(X \times Y) > 2^\omega$.*

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Key words and phrases. extent, product, rectangular, almost discrete, monotonically normal, σ -space, strong σ -space, \mathbb{DC} -like.

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