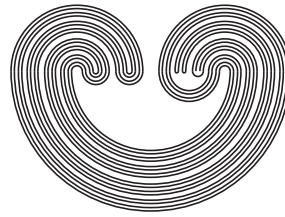


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STRATIFIABILITY AND THE μ -SPACE PROPERTY
OF FUNCTION SPACES
WITH INTERMEDIATE TOPOLOGIES

by

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**STRATIFIABILITY AND THE μ -SPACE PROPERTY
OF FUNCTION SPACES
WITH INTERMEDIATE TOPOLOGIES**

KENICHI TAMANO

Dedicated to the memory of Prof. Phillip L. Zenor

ABSTRACT. We begin to investigate the stratifiability and the μ -space property of intermediate topologies between the topologies of $C_p(X)$ and $C_k(X)$, for a separable metrizable space X . In particular, for the space \mathbb{P} of irrational numbers, we show the following:

(1) There is a family \mathcal{K} of compact sets of \mathbb{P} such that $C_{\mathcal{K}}(\mathbb{P})$ is an M_1 -space and the topology of $C_{\mathcal{K}}(\mathbb{P})$ is strictly between that of $C_p(X)$ and that of $C_k(\mathbb{P})$.

(2) For any nonzero natural number n , let \mathcal{K} be the family of all compact sets with scattered height $< n$. Then $C_{\mathcal{K}}(\mathbb{P})$ is neither a stratifiable space nor a μ -space.

1. INTRODUCTION

All spaces are assumed to be regular T_1 .

A space is a *stratifiable space*, equivalently, an M_3 -space if it has a σ -cushioned pair base. A space is an M_2 -space if it has a σ -closure-preserving quasi-base. Gruenhage [5] and Junnila [11] showed that a space is an M_3 -space if and only if it is an M_2 -space. A space is an M_1 -space if it has a σ -closure-preserving base. A space is F_σ -metrizable if it is a countable union of closed metrizable subspaces. A space is a μ -space

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Key words and phrases. Stratifiable space, M_3 -space, M_1 -space, μ -space, function space, topology of pointwise convergence, compact-open topology.

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