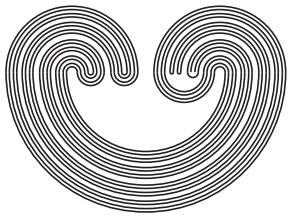
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 61, 2023

Pages 351–398



A homological version of a result of digital topology

by

T. Yung Kong

Electronically published on March 20, 2023

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 61 (2023)
Pages 351-398

E-Published on March 20, 2023

A HOMOLOGICAL VERSION OF A RESULT OF DIGITAL TOPOLOGY

T. YUNG KONG

Abstract. Let $\mathcal{F}$ be any collection of subspaces of a topological space and let $\mathcal{D} \subseteq \mathcal{F}$. We say $\mathcal{D}$ is *homology-simple* in $\mathcal{F}$ if the inclusion map of $\bigcup(\mathcal{F} \setminus \mathcal{D})$ in $\bigcup \mathcal{F}$ induces homology group isomorphisms in all dimensions, and say $\mathcal{D}$ is *hereditarily* homology-simple in $\mathcal{F}$ if every subcollection of $\mathcal{D}$ is homology-simple in $\mathcal{F}$.

Thinning algorithms are used in image processing to simplify binary images. An nD binary image may be regarded as a representation of a finite collection of closed n-cubes that are grid cells of an nD Cartesian grid. Then the goal of thinning is to reduce such a collection of grid cells to a subcollection that is a "thin skeleton" of the collection. Thinning algorithms are commonly designed in such a way that, if $\mathcal{F}_{\text{in}}$ is the original collection of grid cells and $\mathcal{F}_{\text{skel}}$ the resulting skeleton, then $\mathcal{F}_{\text{in}} \setminus \mathcal{F}_{\text{skel}}$ is homology-simple in $\mathcal{F}_{\text{in}}$. We call this the *homology preservation condition*.

For $n \in \{2, 3, 4\}$, a theorem of Bertrand and Couprie implies a local characterization of the hereditarily homology-simple subcollections of any finite collection $\mathcal{F}$ of nD Cartesian grid cells. The theorem and the implied characterization of hereditary homology-simpleness can be used to design good parallel thinning algorithms that automatically satisfy the homology preservation condition, and can also be useful for verifying that a proposed parallel thinning algorithm satisfies that condition.

Bertrand and Couprie's work makes no explicit use of homology. It is based on collapsing of complexes and depends on $\mathcal{F}$ being part of a cubical complex of dimension ≤ 4. This paper shows how

2020 *Mathematics Subject Classification.* Primary 54H30, 55N99; Secondary 68U03, 68U10.

Key words and phrases. hereditarily homology-simple, acyclic space, critical kernel, P-simple, binary digital image, parallel thinning, skeletonization, Khalimsky n-space.