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TOPOLOGY PROCEEDINGS

Volume 62, 2023

Pages 151–161

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by

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Electronically published on April 25, 2023

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
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Volume 62 (2023)
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ELEMENTARY SUBMODELS, CODING STRATEGIES, AND AN INFINITE REAL NUMBER GAME

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Abstract. Matthew Baker [Math. Mag. 80 (2007), no. 5, pp. 377–380] investigates an elegant infinite-length game that may be used to study subsets of real numbers. We present two accessible examples of how an important technique from set theory or a different technique from infinite game theory may be used to answer Baker's question on whether this game provides a precise characterization for countable subsets of real numbers, and we connect this game to the well-studied Banach–Mazur game from topology.

1. Introduction

Let $\omega = \{0, 1, 2, \dots\}$ denote the first infinite ordinal. The following game was investigated by Matthew Baker in [1], based upon a game appearing in [4, p. 67, 1542].

Definition 1.1. Let W be a set of real numbers, called the *payoff set*. During each round $n < \omega$ of the *Cantor game* (denoted $CG(W)$), Alice chooses a legal real number a_n, followed by Bob choosing a legal real number b_n, where a number is legal provided it is strictly greater than all previous choices of Alice and strictly less than all previous choices of Bob.

After ω-many rounds, Alice is said to have won the game provided $\lim_{n\to\infty} a_n \in W$; Bob wins otherwise.

Definition 1.2. A strategy for a player in a game is said to be *winning* provided any counter-strategy for the opponent is defeated by it. That

2020 *Mathematics Subject Classification.* Primary 91A44; Secondary 03C98.
Key words and phrases. elementary submodels, infinite-length games, strategies.