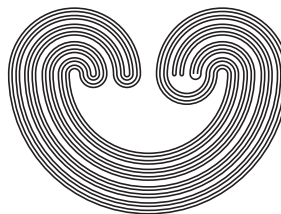


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by

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ELEMENTARY SUBMODELS, CODING STRATEGIES, AND AN INFINITE REAL NUMBER GAME

WILL BRIAN AND STEVEN CLONTZ

ABSTRACT. Matthew Baker [Math. Mag. 80 (2007), no. 5, pp. 377–380] investigates an elegant infinite-length game that may be used to study subsets of real numbers. We present two accessible examples of how an important technique from set theory or a different technique from infinite game theory may be used to answer Baker's question on whether this game provides a precise characterization for countable subsets of real numbers, and we connect this game to the well-studied Banach–Mazur game from topology.

1. INTRODUCTION

Let $\omega = \{0, 1, 2, \dots\}$ denote the first infinite ordinal. The following game was investigated by Matthew Baker in [1], based upon a game appearing in [4, p. 67, 1542].

Definition 1.1. Let W be a set of real numbers, called the *payoff set*. During each round $n < \omega$ of the *Cantor game* (denoted $CG(W)$), Alice chooses a legal real number a_n , followed by Bob choosing a legal real number b_n , where a number is legal provided it is strictly greater than all previous choices of Alice and strictly less than all previous choices of Bob.

After ω -many rounds, Alice is said to have won the game provided $\lim_{n \rightarrow \infty} a_n \in W$; Bob wins otherwise.

Definition 1.2. A strategy for a player in a game is said to be *winning* provided any counter-strategy for the opponent is defeated by it. That

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